AP Calculus AB – Unit 0

Dan the Tutor
Learn by Doing

If you want to do really well in calculus (I mean crush it), you need to have a strong foundation in math. If you don't know Pre-calc, how are you supposed to learn Calc?

The questions and topics in this lesson are the most common math skills you'll need for calculus. They're also the most common mistakes I see. I don't know whether to blame the students or the teachers for your lack of math proficiency, but I do know this – you kids *suck* at math, and it's *my* job to fix that.

1. Factor the expressions:

a) $x^2 + 5x + 4$

1, 4 2, 2

$(x+4)(x+1)$

b) $x^2 - 4x - 12$

1, 12 2, 6 3, 4

$(x-6)(x+2)$

c) $2x^2 - x - 1$

1, 2

$(2x+1)(x-1)$

d) $-x^2 + 3x + 4$

1, 4 2, 2

$(-x+4)(x+1)$

2. FOIL the expressions and then simplify

a) $(x + 14)(x - 2)$

$= x^2 + 14x - 2x - 28$

$= x^2 + 12x - 28$

b) $(2x - 3)(5x + 1)$

$= 10x^2 - 15x + 2x - 3$

$= 10x^2 - 13x - 3$

c) $(x^2 - 1)(4x + 6)$

$= 4x^3 + 6x^2 - 4x - 6$

d) $(3x + 1)(x^2 + 6x - 5)$

$= 3x^3 + 18x^2 - 15x + x^2 + 6x - 5$

$= 3x^3 + 19x^2 - 9x - 5$

3. Simplify the expressions:

a) $\frac{x+1}{2} \cdot \frac{12}{x^2-1} = \frac{(x+1)\cdot 12}{2(x+1)(x-1)}$

$= \frac{(x+1)\,12 \cdot 6}{2(x+1)(x-1)}$

$= \frac{6}{x-1}$

b) $\frac{\frac{5x}{2}}{\frac{2}{5}} = \frac{5x}{2}\left(\frac{5}{2}\right)$

$= \frac{25x}{4}$

c) $\frac{4+x}{x} \cdot \frac{2x^2}{8+2x} = \frac{(4+x)(2x^2)}{x(2(4+x))}$

$= \frac{(4+x)(2x^2)}{x(2)(4+x)}$

$= x$

d) $\frac{\frac{3x}{x-1}}{\frac{x^4}{1-x}} = \frac{3x}{x-1}\left(\frac{1-x}{x^4}\right)$

$= \frac{3x(-(x-1))}{(x-1)x^4}$

$= \frac{-3x(x-1)}{(x-1)x^{4\,3}}$

$= \frac{-3}{x^3}$

e) $\frac{x}{3} + \frac{10-x}{5} = \frac{x}{3}\left(\frac{5}{5}\right) + \frac{10-x}{5}\left(\frac{3}{3}\right)$

$= \frac{5x}{15} + \frac{30-3x}{15}$

$= \frac{30+2x}{15}$

f) $\frac{2+x}{x} + \frac{4x}{x+6} =$

$= \frac{2+x}{x}\left(\frac{x+6}{x+6}\right) + \frac{4x}{x+6}\left(\frac{x}{x}\right)$

$= \frac{2x+x^2+6x+12}{x^2+6x} + \frac{4x^2}{x^2+6x}$

$= \frac{5x^2+8x+12}{x^2+6x}$

4. Write the equation for the line that passes through the points (1,6) and (3,2) …

a) in slope-intercept form ($y = mx + b$)

$x_1 \quad y_1 \quad x_2 \quad y_2$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{2-6}{3-1}$

$= \frac{-4}{2} = -2 \leftarrow m$

$y = -2x + b$

$6 = -2(1) + b$

$6 + 2 = b$

$b = 8$

$y = -2x + 8$

b) in point-slope form ($y - y_1 = m(x - x_1)$)

$m = -2 \quad y_1 = 6 \quad x_1 = 1$

$y_2 = 2 \quad x_2 = 3$

$y - 6 = -2(x - 1)$

Note: $y - 2 = -2(x - 3)$ is also correct

c) Write the equation of the line perpendicular to your answer from part (a) that passes through the point (1,6). Write your answer in slope-intercept form.

perpendicular $\Rightarrow$ slope $m = \frac{1}{2}$ (opposite reciprocal of -2)

$y = \frac{1}{2}x + b$

$6 = \frac{1}{2}(1) + b$

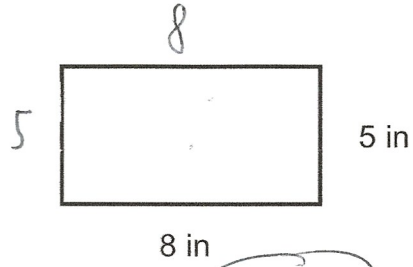
$6 - \frac{1}{2} = b$

$b = \frac{11}{2}$

$y = \frac{1}{2}x + \frac{11}{2}$

5. Find the area and perimeter:

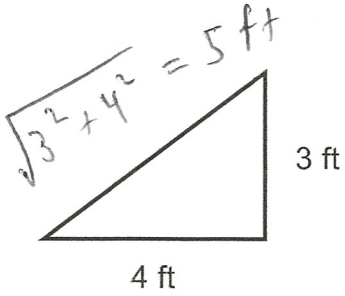
a)



$A = 8 \cdot 5 = 40 \text{ in}^2$

$P = 5 + 5 + 8 + 8 = 26 \text{ in}$

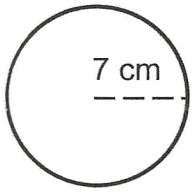
b)



$A = \frac{1}{2}bh = \frac{1}{2}(4)(3) = 6 \text{ ft}^2$

$P = 4\text{ft} + 3\text{ft} + 5\text{ft} = 12 \text{ ft}$

c)



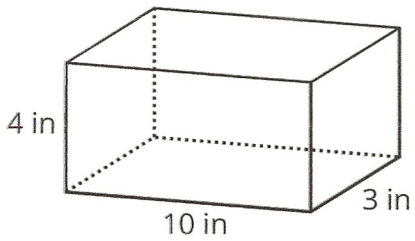
$A = \pi r^2 = \pi(7)^2 = 49\pi$ or 153.9 cm^2

$P = 2\pi r = 2\pi(7) = 14\pi$ or 44.0 cm

(circumference)

6. Find the volume and surface area:

a)



$V = l \cdot w \cdot h = 10(3)(4) = 120 \text{ in}^3$ ← volume

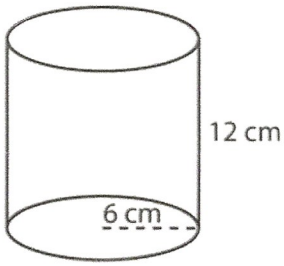
$SA = 2lw + 2lh + 2wh$

$= 2(10)(3) + 2(10)(4) + 2(3)(4)$

$= 60 + 80 + 24$

$= 164 \text{ in}^2$ surface area

b)



$V = \pi r^2 h = \pi(6)^2(12)$

$= 432\pi$ or 1357cm^3 ← volume

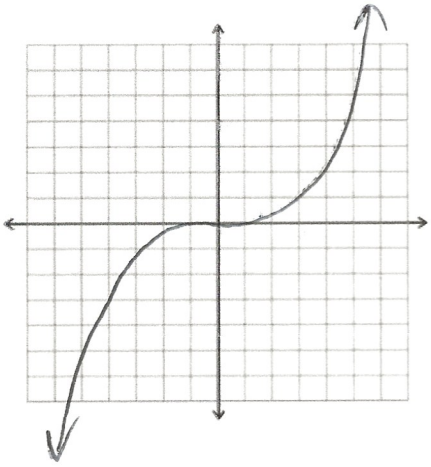
$SA = 2\pi rh + 2\pi r^2$

$= 2\pi(6)(12) + 2\pi(6)^2$

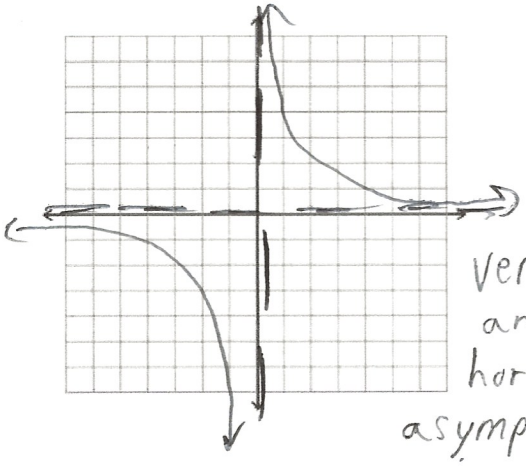
$= 216\pi$ or 678.6 cm^2 surface area

7. Draw the graph for the following functions (just looking for general shape, not numbers)

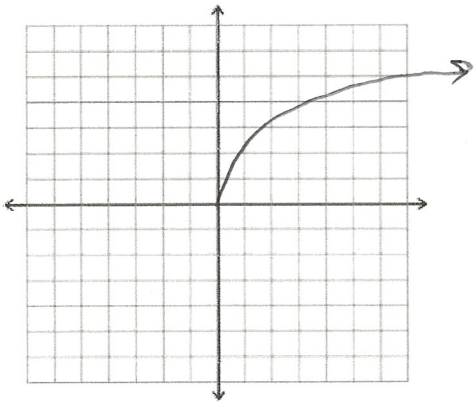
a) $y = x^3$



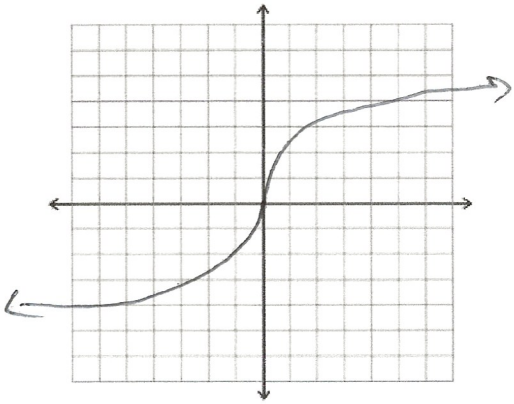
b) $y = \frac{1}{x}$



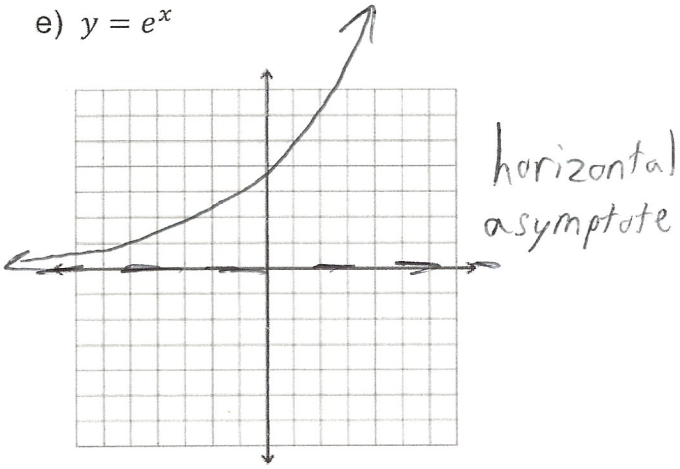
c) $y = \sqrt{x}$



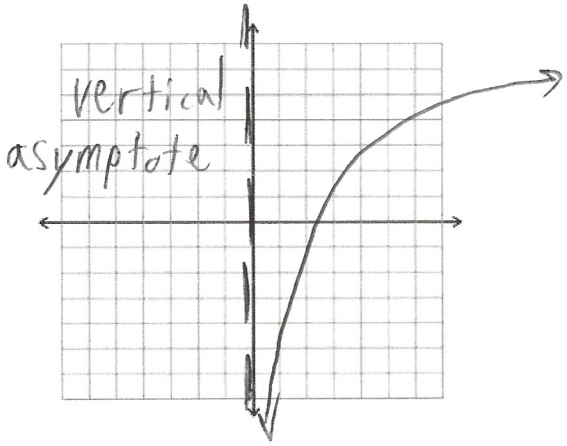
d) $y = \sqrt[3]{x}$



e) $y = e^x$



f) $y = \ln x$



8. Find the Horizontal and Vertical Asymptotes:

a) $y = \frac{x+2}{x^2-4x+3}$ ← Degree of denominator is higher

⇒ Horiz asymp at $y = 0$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 3, 1$

Vertical asymptotes at $x = 3$ and $x = 1$

b) $y = \frac{3x^2-6x}{5x+1}$ ← Degree of numerator is higher

⇒ No horizontal asymptotes

$5x + 1 = 0$

$5x = -1$

$x = \frac{-1}{5}$

Vertical asymptote at $x = -\frac{1}{5}$

c) $y = \frac{2x^2+x+9}{5x^2-8}$ ← Degrees are equal

⇒ look at leading coefficients

Horizontal asymptote at $y = \frac{2}{5}$

$5x^2 - 8 = 0$

$5x^2 = 8$

$x^2 = \frac{8}{5}$

$x = \pm\sqrt{\frac{8}{5}} = \pm\frac{2\sqrt{2}}{5}$

Vertical asymptotes at $x = \frac{2\sqrt{2}}{5}$ and $-\frac{2\sqrt{2}}{5}$

9. Verify the following identities:

Pythagorean Identity

a) $\tan x \sin x + \cos x = \sec x$

$= \frac{\sin x}{\cos x}(\sin x) + \cos x$

$= \frac{\sin^2 x}{\cos x} + \cos x\left(\frac{\cos x}{\cos x}\right)$

$= \frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\cos x}$

$= \frac{1}{\cos x}$

$= \sec x$ ✓

b) $\frac{\tan^2 x}{\tan^2 x + 1} = \sin^2 x$ $\quad \tan^2 x + 1 = \sec^2 x$

$= \frac{\tan^2 x}{\sec^2 x}$

$= \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}}$

$= \frac{\sin^2 x}{\cos^2 x}\left(\frac{\cos^2 x}{1}\right)$

$= \sin^2 x$ ✓

c) $\tan x \cot x - \frac{\sec^2 x}{\tan^2 x} = -\cot^2 x$

$= \frac{\sin x}{\cos x}\left(\frac{\cos x}{\sin x}\right) - \frac{\frac{1}{\cos^2 x}}{\frac{\sin^2 x}{\cos^2 x}}$

$= 1 - \frac{1}{\cos^2 x}\left(\frac{\cos^2 x}{\sin^2 x}\right)$

$= 1 - \frac{1}{\sin^2 x}$

$= 1 - \csc^2 x$

$= -(\csc^2 x - 1)$

$= -\cot^2 x$ ✓

10. Simplify or expand the exponential and logarithmic expressions

a) $\frac{e^3 e^2}{e^7}$

$= e^{3+2-7}$

$= e^{-2}$ or $\frac{1}{e^2}$

b) $\frac{(x^2)^4}{x^{-3}}$

$= \frac{x^8}{x^{-3}}$

$= x^{8-(-3)}$

$= x^{11}$

c) $e^0 - \ln(e)$

$= 1 - 1$

$= 0$

d) $\ln((2x+12)(x-4))$

$= \ln(2x+12) + \ln(x-4)$

e) $\ln\left(\frac{x(3x+5)}{2x-8}\right)$

$= \ln(x) + \ln(3x+5) - \ln(2x-8)$

f) $\ln\left(\frac{(-3x-4)^3}{(x+1)^7}\right)$

$= \ln(-3x-4)^3 - \ln(x+1)^7$

$= 3\ln(-3x-4) - 7\ln(x+1)$