

Practice Problems | Differential Equations and Volume

Separation of Variables

1. $\frac{dy}{dx} = \frac{x+y}{y^2}$

2. $\frac{dy}{dx} = \sqrt[3]{\frac{y}{1-x}}$

Particular Solutions

3. $y' = 2e^x - \cos x$ given $y(0) = 3$

4. Solve the initial value problem
 $h'(t) = 3^t - 6t; h(t) = 2$

5. $\frac{dy}{dt} = y + 1$ given $y(0) = 4$

6. $\frac{dy}{dx} = \frac{x}{y}$ given $y(0) = -2$

7. $\frac{dy}{dx} = x - xy$ given $y(1) = 3$

8. Find dy for $y = e^{3x} + 5x$

Euler's Method

9. Given $\frac{dy}{dx} = x + y, y(0) = 2$,
use Euler's method with 3
equal steps $\Delta x = 1$

10. Given $\frac{dy}{dx} = 2x - y, y(0) = 4$
and approximate
 $y(-1)$ with $\Delta x = -1/2$

11. Given $\frac{dy}{dx} = 3x + 2y + 1$ and
 $f(0) = -2$, starting at $x = 0$
and a step size of $1/2$,
approximate $f(1)$

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Area between curves

Find the area of the region bounded by:

12. $y = x^2, y = 0, x = 2$

13. $f(x) = 2 - x^2, g(x) = 0$

14. $f(x) = 3x^2 - x^2 - 10x,$
 $g(x) = -x^2 + 2x$

15. $y = x^3, y = x$

Using y as the variable

Find the area equation of the region bounded by:

16. $y^2 = 3 - x, y = x - 1$

17. $y^2 - 4x = 4, 4x - y = 16$

18. $y^2 + x = 0, 3y^2 + x = 2$

Disc Method

Find the volume equation:

19. Region: $y = x^2, y = 0, x = 2$
Axis: x-axis

20. Region: $y = x^2, y = 0, x = 2$
Axis: x=2

21. Region: $y = x^2, y = 4$
Axis: y=4

22. Region: $y = x, y = -x, x = -3$
Axis: x=-3

Washer Method

Find the Volume equation:

23. Region: $y = x^2, y = 0, x = 2$
Axis: y-axis

24. Region: $y = x^2, y = 0, x = 2$
Axis: y=5

25. Region: $y = x^2, y = 0, x = 2$
Axis: y=-3

26. Region: $y = x^2, y = 0, x = 2$
Axis: x=4

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Shell Method

Find the volume equation, all examples use the following equations: $y = x^2, y = 0, x = 2$

27. Axis: $x=2$

28. Axis: y -axis

29. Axis: $x=-1$

30. Axis: $x=0$

31. Axis: $y=4$

32. Axis: $y=-2$

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Answer Key

1. $y^3 = \frac{3}{2}x^2 + 12x + C$

2. $y^2 = \left(- (1-x)^{\frac{2}{3}} + C\right)^3$

3. $y = 2e^x - \sin x + 1$

4. $h(t) = \frac{3^t}{\ln 3} - 3t^2 + 5 - \frac{3}{\ln 3}$

5. $y = -1 + 5e^t$

6. $y = -\sqrt{x^2 + 4}$

7. $y = 1 + 2e^{-\frac{1}{2}x^2 + \frac{1}{2}}$

8. $dy = (3e^{3x} + 5)(dx)$

9. $y(3) \approx 20$

10. $y(-1) \approx \frac{19}{2}$

11. $y(1) \approx -\frac{23}{4}$

12. $\frac{8}{3}$

13. $\frac{9}{2}$

14. 24

15. $\frac{1}{2}$

A =

16. $\int_{-2}^1 (3 - y^2 - (y + 1)) dy$

17. $\int_{-4}^5 \left(\frac{1}{4}y + 4 - \left(\frac{1}{4}y^2 - 1\right)\right) dy$

18. $\int_{-1}^1 (-3y^2 + 2 - (-y^2)) dy$

V =

19. $\pi \int_0^2 (x^2 - 0)^2 dx$

20. $\pi \int_0^4 (2 - \sqrt{y})^2 dy$

21. $\pi \int_{-2}^2 (4 - x^2)^2 dx$

22. $\pi \int_0^3 (-y - (-3))^2 dy + \pi \int_{-3}^0 (y - (-3))^2 dy$

23. $\pi \int_0^4 [(2 - 0)^2 - (\sqrt{y} - 0)^2] dy$

24. $\pi \int_0^2 [(5 - 0)^2 - (5 - x^2)^2] dx$

25. $\pi \int_0^2 [(x^2 - (-3))^2 - (0 - (-3))^2]$

26. $\pi \int_0^4 [(4 - \sqrt{y})^2 - (4 - 2)^2] dy$

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Answer Key

V=

$$27. 2\pi \int_0^2 (2-x)(x^2-0)dx$$

$$28. 2\pi \int_0^2 (x)(x^2-0)dx$$

$$29. 2\pi \int_0^2 (x-(-1))(x^2-0)dx$$

$$30. 2\pi \int_0^4 (y)(2-\sqrt{y})dy$$

$$31. 2\pi \int_0^4 (4-y)(2-\sqrt{y})dy$$

$$32. 2\pi \int_0^4 (y+2)(2-\sqrt{y})dy$$