

Unit 5 Exam

Recommended amount of time: 2 hours

- Topics covered on this exam:
 - Riemann sums
 - Trapezoidal rule
 - Properties of integrals
 - Rules of integration
 - Indefinite and definite integrals
 - U-substitution
 - Fundamental Theorem of Calculus
- This exam is composed of 21 multiple choice questions and 2 free response questions
- Calculators are allowed for computation use only (addition, division, square root, exponents, etc.)
- For the free response section, show all of your work (think about partial credit on an actual test)
- When you are finished with the test, see which questions you got wrong and review those questions





1. If F and f are continuous functions such that $F'(x) = f(x)$ for all x , then $\int_a^b f(x) dx$ is

- (A) $F'(a) - F'(b)$
- (B) $F'(b) - F'(a)$
- (C) $F(a) - F(b)$
- (D) $F(b) - F(a)$
- (E) None of the above

2. If $F(x) = \int_0^x \sqrt{t^3 + 1} dt$, then $F'(2) =$

- (A) -3
- (B) -2
- (C) 2
- (D) 3
- (E) 18

3. The expression $\frac{1}{20} \left(\sqrt{\frac{1}{20}} + \sqrt{\frac{2}{20}} + \sqrt{\frac{3}{20}} + \cdots + \sqrt{\frac{20}{20}} \right)$ is a Riemann sum approximation for

- (A) $\int_0^1 \sqrt{\frac{x}{20}} dx$
- (B) $\int_0^1 \sqrt{x} dx$
- (C) $\frac{1}{20} \int_0^1 \sqrt{\frac{x}{20}} dx$
- (D) $\frac{1}{20} \int_0^1 \sqrt{x} dx$
- (E) $\frac{1}{20} \int_0^{20} \sqrt{x} dx$

AP Calculus AB – Unit 5



6. $\int_0^3 (4x - 3) dx$

(A) 0

(B) 9

(C) 15

(D) 20

(E) 29

7. $\int_0^8 \frac{dx}{\sqrt{1+x}} =$

(A) $4\sqrt{2}$

(B) $\frac{3}{2}$

(C) 2

(D) 4

(E) 6

AP Calculus AB – Unit 5



8. $\int \frac{x^2}{e^{x^3}} dx =$

(A) $-\frac{1}{3} \ln e^{x^3} + C$

(B) $-\frac{e^{x^3}}{3} + C$

(C) $-\frac{1}{3e^{x^3}} + C$

(D) $\frac{1}{3} \ln e^{x^3} + C$

(E) $\frac{x^3}{3e^{x^3}} + C$

9. $\int_0^{\frac{\pi}{2}} 2 \sec^2\left(\frac{x}{2}\right) dx =$

(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) 2

(E) 4

AP Calculus AB – Unit 5



10. $\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{2 \cos(2x)}{\sin(2x)} dx =$

- (A) $-\ln \frac{\sqrt{2}}{2}$ (B) $\ln \frac{1}{2}$ (C) $\ln \sqrt{3}$ (D) $\ln \frac{\sqrt{3}}{2}$ (E) $\ln e$

11. If f is a linear function and $0 < a < b$, then $\int_a^b f''(x) dx =$

- (A) 0 (B) 1 (C) $\frac{ab}{2}$ (D) $b - a$ (E) $\frac{b^2 - a^2}{2}$

AP Calculus AB – Unit 5



12. $\int_1^e \left(\frac{x^2 - 1}{x} \right) dx =$

- (A) $e - \frac{1}{e}$ (B) $e^2 - e$ (C) $\frac{e^2}{2} - e + \frac{1}{2}$ (D) $e^2 - 2$ (E) $\frac{e^2}{2} - \frac{3}{2}$

13. What are all values of k for which $\int_{-3}^k x^2 dx = 0$?

- (A) -3 (B) 0 (C) 3 (D) -3 and 3 (E) $-3, 0,$ and 3

AP Calculus AB – Unit 5



Use the following information for questions 14-18. Let $\int_0^3 f(x) = 2$, $\int_0^6 f(x) = 8$, and $\int_3^6 g(x) = 3$

14. $\int_3^6 f(x) dx =$

- (A) 2 (B) 8 (C) -8 (D) 6 (E) Cannot be determined

15. $\int_6^3 3g(x) dx =$

- (A) -3 (B) 9 (C) -9 (D) 3 (E) Cannot be determined

16. $\int_0^3 2(f(x) + 1) dx =$

- (A) 6 (B) 5 (C) 4 (D) 10 (E) Cannot be determined

17. $\int_3^6 (f(x) * g(x)) dx =$

- (A) 3 (B) 8 (C) -1 (D) 5 (E) Cannot be determined

18. $\int_1^1 3g(x) dx =$

- (A) 0 (B) 3 (C) 9 (D) -3 (E) Cannot be determined



19. Given $f(x) = \sec^2 x + \frac{1}{\pi}$ and $F(\pi) = 4$, where $F(x) = \int f(x) dx$. What is $\int f(x) dx$?

(A) $-\cos x + \frac{x}{\pi} + 2$

(B) $\cos x + \frac{x}{\pi} + 4$

(C) $\tan x + \frac{x}{\pi} + 3$

(D) $\sin x + \frac{x}{\pi} + 3$

(E) $-\sin x + \frac{x}{\pi} + 3$

20. What is the value of k for which $\int_0^k \frac{3}{1+x^2} dx = \frac{3\pi}{4}$?

(A) 0

(B) 1

(C) 2

(D) -3

(E) 4



21. $\int \frac{-4e^{4/x}}{x^2} dx =$

(A) $\frac{-4e^{4/x}}{x^2} + C$

(B) $-4e^{4/x} + C$

(C) $e^{4/x} + C$

(D) $\frac{e^{4/x}}{x} + C$

(E) $4e^4 - e^x + C$



Free Response

Time t (seconds)	0	10	20	30	40	50	60	70	80
Velocity $v(t)$ (m/s)	24	32	44	82	118	164	222	286	348

- A bullet train is accelerating and travelling in the positive direction. The velocity $v(t)$ is recorded for selected values of t over the interval $0 \leq t \leq 80$ seconds, as shown in the table above. Use this data to answer the following questions about the bullet train.

 - Approximate the total distance travelled over the first minute, $\int_0^{60} v(t) dt$, using a left Riemann sum with **3 subintervals of equal length**. Include proper units. Is this an overestimate or underestimate?
 - Approximate the total distance travelled over the first minute, $\int_0^{60} v(t) dt$, using a right Riemann sum with **3 subintervals of equal length**. Include proper units. Is this an overestimate or underestimate?



- c. Approximate the total distance travelled over the first minute, $\int_0^{60} v(t) dt$, using a midpoint Riemann sum with **3 subintervals of equal length**. Include proper units.
- d. Approximate the total distance travelled over the **first 80 seconds**, $\int_0^{80} v(t) dt$, using the trapezoidal method with **4 subintervals of equal length**. Include proper units.



2. The functions f and g are given by $f(x) = \int_2^x (4t - t^2) dt$ and $g(x) = f(e^x)$.
- a. Find $f'(x)$ and $g'(x)$.

- b. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 2$

AP Calculus AB – Unit 5

Dan the Tutor



Learn by Doing

- c. Function h is defined by $h(x) = \int_0^{x^2} \left(\frac{2}{\sqrt{t}} - 1 \right) dt$. What is the x -value of the relative maximum of $h(x)$?