Simple Harmonic Motion Problems

1. A spring with spring constant 100 N/m is pulled back 5 meters. What is the spring's potential energy?

2. A 3 kg mass is launched from a spring that had 600 J of potential energy stored in it. What is the maximum speed of the mass?

3. A 12 kg mass connected to a spring oscillates back and forth. The mass achieves a maximum speed of 4 m/s when it passes the spring's rest length. If the spring constant is 150 N/m, what is the amplitude of the mass (in other words, how far can the spring be stretched)?

4. A mass undergoes simple harmonic motion with a spring. When the spring is 30 cm from its rest length, the velocity is 3.5 m/s. The mass is 2 kg and the spring constant is 100 N/m. What is the maximum displacement of the spring? What is the maximum speed of the mass?

5. A mass (680 grams) oscillates back and forth on a spring (k = 250 N/m). If the amplitude of the system is 1 meter, what is the mass' speed when 50% of the energy is in the form of kinetic energy?

6. A block of mass 6.3 kg is in simple harmonic motion with a spring of spring constant 325 N/m. If the amplitude of the system is 78 cm, what is the maximum force on the block (from the spring)? At that instant, what is the magnitude of the block's acceleration?

7. The period of a simple harmonic motion system is 800 ms. What is its frequency?

8. The frequency of a spring-mass system is 2 Hz. If the spring constant is 225 N/m, what is the mass?

9. A 30 kg mass is suspended from a pendulum and allowed to swing back and forth. If the pendulum is 1.8 meters long, what is the period of the pendulum?

10. A pendulum has a frequency of 4.2 Hz on Earth. If the pendulum is moved to the moon where the gravity is 1/6 that of Earth, what will be its new frequency?

1. $U_s = \frac{1}{2}kx^2$

$= \frac{1}{2}(100)(5)^2 = 1{,}250\ J$

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2. $E_{tot,1} = E_{tot,2}$

$E_{tot,1}$: All potential

$E_{tot,2}$: All kinetic (max speed)

$U_s = K$

$600 = \frac{1}{2}(3)v^2$

$400 = v^2$

max speed → $v = 20\ m/s$

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3. $E_{tot,1} = E_{tot,2}$

$E_{tot,1}$: All kinetic (max speed)

$E_{tot,2}$: All potential (amplitude)

$K = U_s$

$\frac{1}{2}(12)(4)^2 = \frac{1}{2}(150)A^2$ ← A = Amplitude

$96 = 75A^2$

$1.28 = A^2$

$A = 1.13\ m$

4. Finding max displacement

$E_{tot,1} = E_{tot,2}$

$U_{s,1} + K_1 = U_{s,2}$

$\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA^2$ ← amplitude

30 cm = .30 m

$\frac{1}{2}(100)(.30)^2 + \frac{1}{2}(2)(3.5)^2 = \frac{1}{2}(100)A^2$

$16.75 = 50A^2$

$.335 = A^2$

$A = .58\,m$ or $58\,cm$ ← Amplitude

Finding max speed

$E_{tot,1} = E_{tot,2}$

$U_{s,1} + K_1 = K_2$

$\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}mv_{max}^2$

same number as before

$16.75 = \frac{1}{2}(2)v_{max}^2$

$16.75 = v_{max}^2$

$v_{max} = 4.09\,m/s$ ← Max speed

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5. $E_{tot} = \frac{1}{2}kA^2$

$= \frac{1}{2}(250)(1)^2$

$E_{tot} = 125\,J$

when 50% of E_{tot} is kinetic

$K = .5(125) = \frac{1}{2}mv^2$

$62.5 = \frac{1}{2}(.680)v^2$

680 g = .680 kg

$183.8 = v^2$

$v = 13.6\,m/s$

6. The max force (and acceleration) occurs when the spring is at its max ~~stre~~ displacement.

$F_{spring} = kx$ $\quad 78\text{ cm} = .78\text{ m}$

$= 325(.78) = \boxed{253.5\text{ N}}$ ← max force

$F = ma$

$253.5 = 6.3a$ $\quad \boxed{a = 40.2\ \text{m/s}^2}$ ← max acceleration

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7. $T = \frac{1}{f}$ $\quad 800\text{ ms} = .8\text{ s}$ $\quad f = \frac{1}{T} = \frac{1}{.8} = \boxed{1.25\text{ Hz}}$

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8. $T = 2\pi\sqrt{\frac{m}{k}}$, $f = \frac{1}{T}$

$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$2 = \frac{1}{2\pi}\sqrt{\frac{225}{m}}$

Multiply 2π on both sides

$4\pi = \sqrt{\frac{225}{m}}$

square both sides →

$16\pi^2 = \frac{225}{m}$

$16\pi^2 m = 225$

$m = \frac{225}{16\pi^2}$

$\boxed{m = 1.42\text{ kg}}$

9. Mass doesn't matter for pendulums

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{1.8}{9.8}} = 2.69\text{ s}$$

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10. First, find the length of the pendulum (while on Earth)

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

$$4.2 = \frac{1}{2\pi}\sqrt{\frac{9.8}{L}}$$

$$4.2(2\pi) = \sqrt{\frac{9.8}{L}}$$

$$(26.39)^2 = \frac{9.8}{L}$$

$$L(26.39)^2 = 9.8$$

$$L = .0141\text{ m}$$

(length doesn't change on the moon)

Now, we find the new frequency

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

$$g = \frac{9.8}{6} = 1.63$$

$$f = \frac{1}{2\pi}\sqrt{\frac{1.63}{.0141}}$$

$$f = 1.71\text{ Hz}$$