Rotational Kinetic Energy Problems

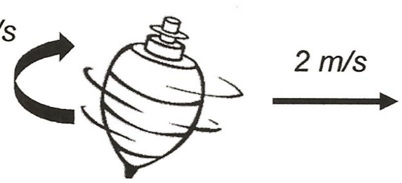
1. Calculate the rotational kinetic energy of an object revolving at 9 rad/s with a moment of inertia of 2 kg·m^2.

2. A wheel spins at 3000 rpm (rotations per minute). The wheel can be treated as a solid disk with mass 5 kg and radius 80 cm. What is the rotational kinetic energy of the wheel?

3. A man whirls around a heavy ball on a rope around his body. The rotational kinetic energy is 1,950 J when the angular velocity is 11 rad/s. Since the mass of the rope is negligible, the ball can be treated as a point source of mass 1 meter away from his body. What is the mass of the ball?

4. A long bar hinged at its center is spinning with 256 J of rotational kinetic energy. If the bar is 6 kg and has a length of 4 m, what is the tangential velocity at the end of the rod?

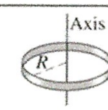
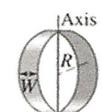
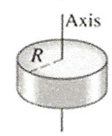
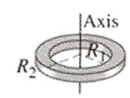
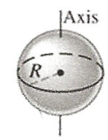
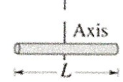
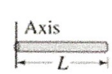
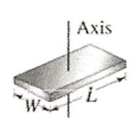
5. You spin a top so that it is both rotating and moving. It spins at a rate of 5 rotations per second while moving to the side at a speed of 2 m/s. If the mass of the top is 500 grams and its moment of inertia is 0.001 kg·m^2, what is the total kinetic energy in the system?



6. A 4 kg metal hoop is rolling down a 10-meter-tall hill. At the bottom, 60% of the hoop's energy is in the form of translational kinetic energy while the rest is rotational kinetic energy. What is the rotational kinetic energy of the hoop at the bottom of the hill?

7. A can of soup (solid disk) rolls down a 5-meter-long ramp at an elevation of 23°. The can has a mass of 0.8 kg and radius 6 cm. If the can starts from rest, what will be its speed at the bottom?

for your reference

	Object	Location of axis		Moment of inertia
(a)	**Thin hoop,** radius R	Through center		MR^2
(b)	**Thin hoop,** radius R width W	Through central diameter		$\frac{1}{2}MR^2 + \frac{1}{12}MW^2$
(c)	**Solid cylinder,** radius R	Through center		$\frac{1}{2}MR^2$
(d)	**Hollow cylinder,** inner radius R_1 outer radius R_2	Through center		$\frac{1}{2}M(R_1^2 + R_2^2)$
(e)	**Uniform sphere,** radius R	Through center		$\frac{2}{5}MR^2$
(f)	**Long uniform rod,** length L	Through center		$\frac{1}{12}ML^2$
(g)	**Long uniform rod,** length L	Through end		$\frac{1}{3}ML^2$
(h)	**Rectangular thin plate,** length L, width W	Through center		$\frac{1}{12}M(L^2 + W^2)$

1. $K_r = \frac{1}{2} I \omega^2$ $I = 2$ $\omega = 9$

$= \frac{1}{2}(2)(9)$

$= 9\ J$

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2. $K_r = \frac{1}{2} I \omega^2$ $\omega = 3000 \frac{rot}{min}\left(\frac{2\pi\ rad}{1\ rot}\right)\left(\frac{1\ min}{60\ s}\right)$

$\omega = 314.2\ rad/s$

$I_{disk} = \frac{1}{2} m r^2$ $r = 80\ cm = .80\ m$

$= \frac{1}{2}(5)(.80)^2$

$I = 1.6$

$K_r = \frac{1}{2}(1.6)(314.2)^2$

$= 78,957\ J$

3. $K_r = \frac{1}{2} I \omega^2$ $\quad K_r = 1{,}950$ $\quad \omega = 11 \text{ rad/s}$

$I_{\text{point source}} = mr^2 = m(1)^2$

$I = m$

$1950 = \frac{1}{2}(m)(11)^2$

$1950 = 60.5m$ $\quad$ $m = 32.2 \text{ kg}$

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4. $K_r = \frac{1}{2} I \omega^2$ $\quad K_r = 256$ $\quad I_{\text{rod, center}} = \frac{1}{12} m L^2$

$\omega = ?$

$= \frac{1}{12}(6)(4)^2$

$I = 8$

$256 = \frac{1}{2}(8)(\omega)^2$

$256 = 4\omega^2$

$64 = \omega^2$

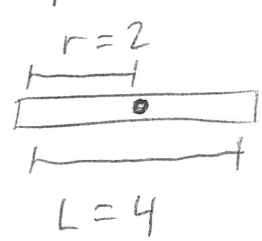
$\omega = 8 \text{ rad/s}$

tangential velocity

$V = \omega \cdot r$

$V = 8(2)$

$V = 16 \text{ m/s}$



5. $K_{tot} = K_T + K_r$

$\omega = 5 \frac{rot}{s}\left(\frac{2\pi\ rad}{1\ rot}\right)$

$\omega = 10\pi \frac{rad}{s}$

$K_T = \frac{1}{2} m v^2$

$m = 500g = .5\ kg$

$K_T = \frac{1}{2}(.5)(2)^2$

$K_T = 1\ J$

$K_r = \frac{1}{2} I \omega^2$

$= \frac{1}{2}(.001)(10\pi)^2$

$K_r = .493\ J$

$K_{tot} = 1 + .493 = 1.493\ J$ ← Total

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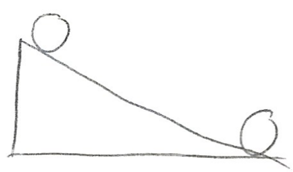
6. ① All potential



② All kinetic

$E_{tot,1} = E_{tot,2}$

$mgh = K_{tot}$

$4(9.8)(10) = K_{tot}$

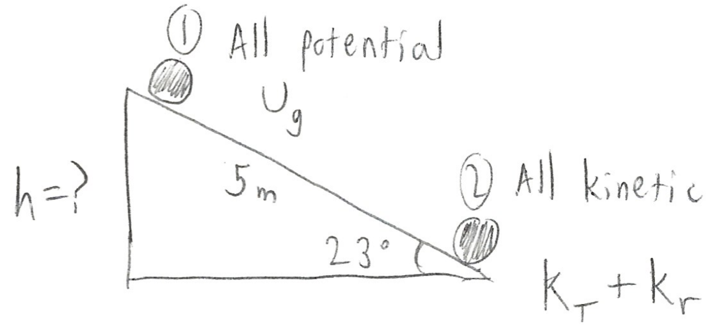
$K_{tot} = 392\ J$

60% translational
40% rotational

$K_r = .4(392)$

$= 156.8\ J$ ← rotational kinetic energy

7.



$$E_{tot,1} = E_{tot,2}$$
$$U_g = K_T + K_r$$

$$\sin 23° = \frac{h}{5} \qquad U_g = mgh$$

$$h = 5\sin 23° \qquad U_g = .8(9.8)(1.954) = 15.32\ J$$

$$h = 1.954\ m$$

$$r = 6\ cm = .06\ m$$

$$K_T = \tfrac{1}{2}mv^2 \qquad v = \omega r = \omega(.06)$$

$$= \tfrac{1}{2}(.8)(\omega(.06))^2$$

$$= \tfrac{1}{2}(.8)(\omega^2)(.06)^2$$

$$K_T = .00144\,\omega^2$$

$$K_r = \tfrac{1}{2}I\omega^2$$

$$I_{disk} = \tfrac{1}{2}mr^2$$

$$= \tfrac{1}{2}(.8)(.06)^2$$

$$I = .00144$$

$$K_r = \tfrac{1}{2}(.00144)\omega^2 = .00072\,\omega^2$$

$$U_g = K_T + K_r$$

$$15.32 = .00144\,\omega^2 + .00072\,\omega^2$$

$$15.32 = .00216\,\omega^2$$

$$7093 = \omega^2$$

$$\omega = 84.2\ \tfrac{rad}{s}$$

$$v = \omega r$$

$$v = 84.2(.06)$$

$$\boxed{v = 5.05\ m/s}$$

speed at the bottom