Unit 2 Exam

Recommended amount of time: 2 hours

- Topics covered on this exam:
  - Rules of differentiation (power rule, product rule, quotient rule, chain rule, etc.)
  - Slope at a point
  - Higher order derivatives
  - Equation of a tangent line
  - Position, velocity, and acceleration
  - Graph of a derivative
- This exam is composed of 18 multiple choice questions and 3 free response questions
- Calculators are allowed for computation use only (addition, division, square root, exponents, etc.)
- For the free response section, show all of your work (think about partial credit on an actual test)
- When you are finished with the test, see which questions you got wrong and review those questions

AP Calculus AB – Unit 2

Dan the Tutor
Learn by Doing

1. A particle moves in a straight line with position given by $p(t) = 2t + 2$ over any time t. What is the velocity of the particle at $t = 2$?

(A) 6 (B) 2 (C) 4 (D) 8 (E) 0

2. If $f(x) = \sin(x)$, then $f'\left(\frac{3\pi}{2}\right) =$

(A) 0 (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) 1 (E) $\sqrt{3}$

3. Find y' to the equation $y = x(3-x)^2$.

(A) $y' = -2x(3-x)$

(B) $y' = (3-x)^2 + 2x(3-x)^2$

(C) $y' = (3-x)^2$

(D) $y' = (3-x)^2 - 2x(3-x)$

(E) $y' = (3-x)^2 + 2x(3-x)$

4. If $y = 2\sin(x) - \cot(x)$, then $\frac{dy}{dx} =$

(A) $2\cos(x) + \sin(x)$
(B) $2\cos(x) - 2\sin(x)$
(C) $\sec(x) + 2\csc(x)$
(D) $2\cos(x) - csc^2(x)$
(E) $2\cos(x) + csc^2(x)$

5. If $f(x) = (2x^2 - 3x + 1)^{4/3}$, then $f'(0)$ is

(A) 0 (B) 1 (C) $-\frac{4}{3}$ (D) -4 (E) $\frac{4}{3}$

6. An equation of the line tangent to the graph of $y = \frac{x+4}{6x-1}$ at the point (1,2) is

(A) $y = -x + 3$
(B) $y = -4x + 6$
(C) $y = x + 1$
(D) $y = 4x - 2$
(E) $y = 6x - 4$

7. If $f(x) = (2x+3)^2 \cos(x)$, then $f'(0) =$

(A) 0 (B) 12 (C) 1 (D) -1 (E) -2

8. The slope of the line normal to the graph of $y = -4\cos(x)$ at $x = \frac{\pi}{6}$ is

(A) -2 (B) -4 (C) 4 (D) $-\frac{1}{2}$ (E) $\frac{1}{2}$

9. What is the second derivative of $y = \frac{2}{x^2}$?

(A) $y'' = \frac{-4}{x^3}$ (B) $y'' = \frac{4}{x}$ (C) $y'' = \frac{12}{x^4}$ (D) $y'' = \frac{4}{x^3}$ (E) $y'' = \frac{-12}{x^4}$

10. At what point on the graph of $y = x^2 - 3$ is the tangent line parallel to the line $y = 4x - 1$?

(A) $(1, -2)$ (B) $(1,3)$ (C) $(2,2)$ (D) $(2,7)$ (E) $(2,1)$

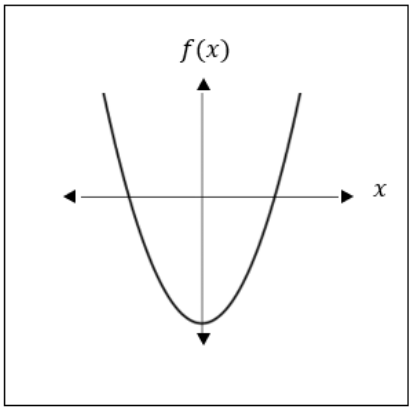
11. If $f(x) = x\sqrt{3x-2}$, then $f'(x) =$

(A) $\frac{9x-4}{2\sqrt{3x-2}}$ (B) $\frac{4-9x}{2\sqrt{3x-2}}$ (C) $\frac{6x-2}{\sqrt{3x-2}}$ (D) $\frac{2-6x}{\sqrt{3x-2}}$ (E) $\frac{6x-2}{2\sqrt{3x-2}}$

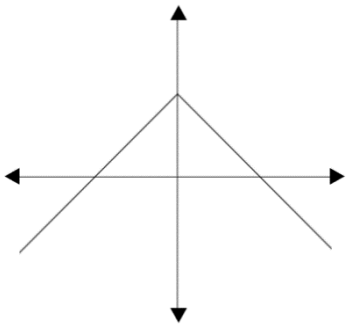
12. What is the slope of $y = \sec\left(\frac{x}{2}\right)$ at $\left(\frac{\pi}{2}, \sqrt{2}\right)$?

(A) $-\sqrt{2}$ (B) $-\frac{\sqrt{2}}{2}$ (C) $\sqrt{2}$ (D) $\frac{\sqrt{2}}{2}$ (E) 0

13. Which of the following could be the derivative of f(x)?



(A)



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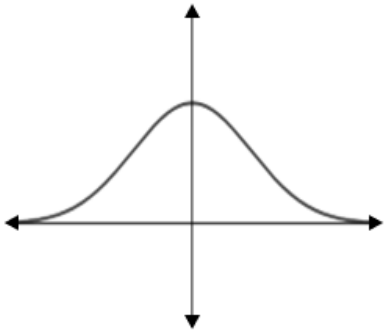

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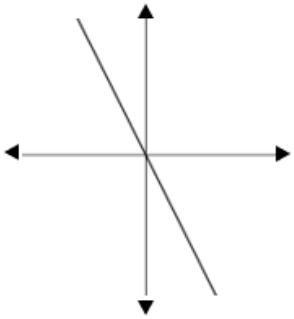

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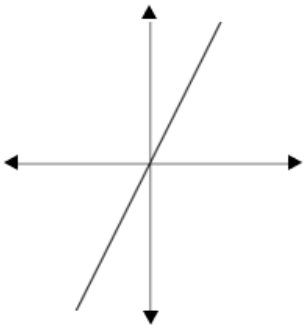

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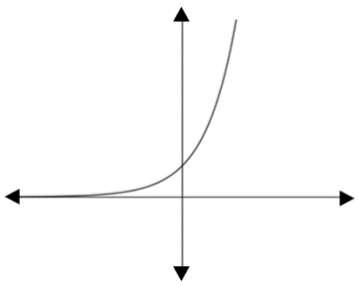


14. If a car's velocity is calculated by the function $f(t) = \frac{3t-2}{t+1}$, what is the acceleration of the car at $t = 4$?

(A) 1 (B) $\frac{31}{25}$ (C) $-\frac{31}{25}$ (D) $\frac{1}{5}$ (E) $-\frac{1}{5}$

15. What is the second derivative of $f(x) = \frac{x^3(1-x)}{2}$ at $x = 2$?

(A) -18 (B) 1 (C) 9 (D) -9 (E) 18

16. What is the slope of the function $f(x) = \cos\left((3x-1)^2\right)$?

(A) $f'(x) = (18x-6)\sin\left((3x-1)^2\right)$

(B) $f'(x) = (6-18x)\sin\left((3x-1)^2\right)$

(C) $f'(x) = (18x-6)\sin(3x-1)$

(D) $f'(x) = (6-18)\sin(3x-1)$

(E) $f'(x) = (2-6x)\sin(3x-1)$

17. What is the derivative of the function $f(x) = -\frac{x^2+10}{1-x}$?

(A) $f'(x) = \frac{x^2-2x-10}{(1-x)^2}$

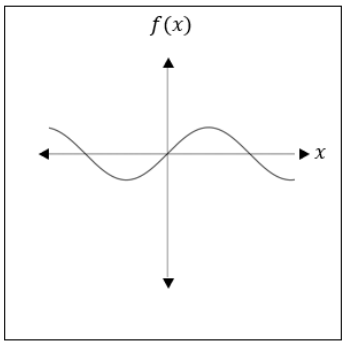
(B) $f'(x) = \frac{-x^2+2x+10}{(1-x)^2}$

(C) $f'(x) = \frac{x^2-2x-10}{1-x}$

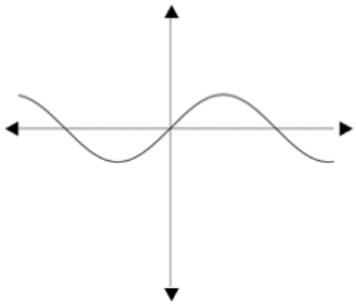
(D) $f'(x) = \frac{-x^2+2x+10}{1-x}$

(E) $f'(x) = \frac{-x^2+2x+10}{2(1-x)}$

18. Which of the following could be the derivative of f(x)?



(A)



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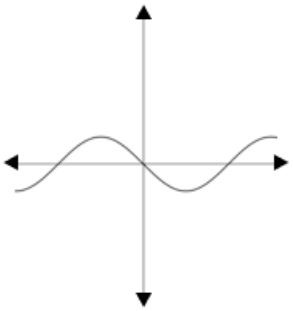

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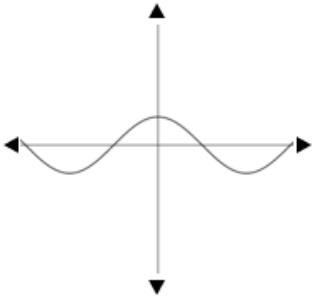

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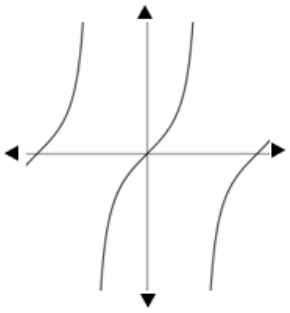

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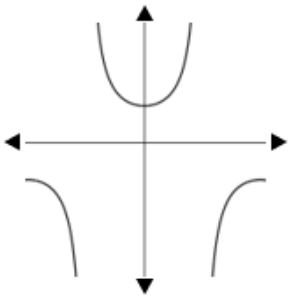

Free Response

1. Given the function $f(x) = \frac{1}{\sqrt{x}}$

   A) What is the equation for the line tangent to f(x) when $x = 1$?

   B) What is the equation for the line normal to f(x) when $x = \frac{1}{4}$?

   C) When is the slope equal to -4? (just give the x-value)

2. The position of particle 1 can be modelled by $x(t) = 5t^3 - 4t^2 + 6t - 2$.

   A) What is the particle's velocity at $t = 3$?

   B) What is the particle's acceleration at $t = 3$?

   C) A different particle (particle 2) is moving along the x-axis with $x(t) = t \cdot \cos(2t)$. Which particle is moving faster at $t = 0$?

3. A spherical water balloon is being filled with water. The volume, in cm^3, of the water balloon can be calculated using the function $V(t) = \frac{4}{3}\pi(t^2 + 5)^3$ where t is time in minutes ($t \geq 0$).

   A) The surface area can be calculated by taking the derivative of the volume. What is the expression of $S(t)$ (surface area of the sphere)?

   B) What is the surface area of the water balloon after 2 minutes of filling?

C) We want the water balloon to have a volume larger than 1000 cm^3 and have the volume equal to the surface area. How long should the water balloon be filled for? (Hint: you should use a calculator for this problem)

D) Below is a graph of the volume of a different water balloon vs time. Draw the graph of the surface area vs time.