



Problem Set 18: Critical Values and Particle Motion

Find the critical points of the function (just the x-values).

1. $y = -4t^2 + 8t - 4$

$$y' = -8t + 8 = 0$$

$$8t = 8$$

$$t = 1$$

2. $y = 3x(x - 1) = 3x^2 - 3x$

$$y' = 6x - 3 = 0$$

$$6x = 3$$

$$x = \frac{3}{6} = \left(\frac{1}{2}\right)$$

3. $f(x) = -5\sqrt{5-x} = -5(5-x)^{1/2}$

$$f'(x) = -5\left(\frac{1}{2}\right)(5-x)^{-1/2}(-1)$$

$$= \frac{-5(-1)}{2(\sqrt{5-x})}$$

$$= \frac{5}{2\sqrt{5-x}} = 0$$

$$2\sqrt{5-x} = 0$$

$$\sqrt{5-x} = 0$$

$$5-x = 0$$

$$x = 5$$

4. $h(x) = \frac{x^2+2}{4x+2} \leftarrow u = x^2+2 \quad u' = 2x$
 $\leftarrow v = 4x+2 \quad v' = 4$

$$h'(x) = \frac{(4x+2)(2x) - (x^2+2)(4)}{(4x+2)^2}$$

$$= \frac{8x^2+4x-4x^2-8}{(4x+2)^2} = 0$$

$$8x^2+4x-4x^2-8=0$$

$$4x^2+4x-8=0$$

$$4(x^2+x-2)=0$$

$$4(x+2)(x-1)=0$$

$$x = -2, 1$$

$$(4x+2)^2 = 0$$

$$4x+2=0$$

$$4x = -2$$

$$x = -\frac{1}{2}$$

3 critical points

AP Calculus AB – Unit 4

Dan the Tutor



Learn by Doing

5. The position of a particle along the y-axis for any time t can be represented by the equation $g(t) = t^4 - 4t^3 + 10$.

- a) What is the velocity of the particle at time $t = 2$?

$$\begin{aligned} \text{velocity} &= g'(t) = 4t^3 - 12t^2 \\ g'(2) &= 4(2)^3 - 12(2)^2 = 4(8) - 12(4) \\ &= 32 - 48 \\ &= \boxed{-16} \end{aligned}$$

- b) What is the acceleration of the particle at time $t = 2$?

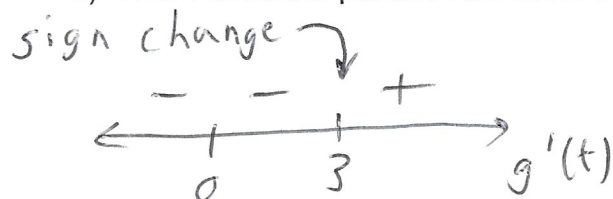
$$\begin{aligned} \text{acceleration} &= g''(t) = 12t^2 - 24t \\ g''(2) &= 12(2)^2 - 24(2) = 48 - 48 \\ &= 12(4) - 48 = \boxed{0} \end{aligned}$$

- c) When does the particle stop?

$$\begin{aligned} \text{Particle stop} &\Rightarrow \text{velocity is zero} \Rightarrow g'(t) = 0 \\ g'(t) &= 4t^3 - 12t^2 = 0 \\ 4t^2(t-3) &= 0 \end{aligned}$$

$$\begin{aligned} 4t^2 &= 0 & t-3 &= 0 \\ t &= 0 & t &= 3 \end{aligned}$$

- d) When does the particle turn around?



Plot critical points on number line

$$\begin{aligned} g'(-1) &= 4(-1)^3 - 12(-1)^2 = -4 - 12 = -16 \text{ (negative)} \\ g'(1) &= 4(1)^3 - 12(1)^2 = 4 - 12 = -8 \text{ (negative)} \\ g'(4) &= 4(4)^3 - 12(4)^2 = 256 - 192 = 64 \text{ (positive)} \end{aligned}$$

Turns around at $t=3$



AP Calculus AB – Unit 4

6. A particle moves along the x-axis so that at time $t \geq 0$ the position of the particle is $x(t) = t \cdot \ln t$

- a) What is the velocity of the particle at time $t = 1$?

$$x'(t) = t\left(\frac{1}{t}\right) + \ln(t) \cdot (1) \leftarrow \text{Product Rule}$$

$$x'(1) = 1\left(\frac{1}{1}\right) + \ln(1) = 1 + 0 = \boxed{1}$$

- b) What is the acceleration of the particle at time $t = 1$?

$$x'(t) = 1 + \ln(t)$$

$$x''(t) = \frac{1}{t} \quad x''(1) = \frac{1}{1} = \boxed{1}$$

- c) When does the particle stop?

$$x'(t) = 1 + \ln(t) = 0$$

$$e^{\ln(t)} = e^{-1}$$

$$t = e^{-1} = \boxed{\frac{1}{e}}$$

- d) When does the particle turn around?

Sign change \swarrow

$$\begin{array}{c} \leftarrow - \quad + \rightarrow \\ \frac{1}{e} \end{array} x'(t)$$

$$x'(t) = 1 + \ln(t)$$

$$\boxed{t = \frac{1}{e}}$$

$$x'(0) = 1 + \ln(0) \leftarrow \text{error (can't choose 0)}$$

$$x'(.1) = 1 + \ln(.1) = 1 - 2.30 \text{ (negative)}$$

$$x'(10) = 1 + \ln(10) = 1 + 2.30 \text{ (positive)}$$

AP Calculus AB – Unit 4

Dan the Tutor



Learn by Doing

7. The position vs. time graph of a particle is shown. The position is measured in meters and the time is measured in seconds.

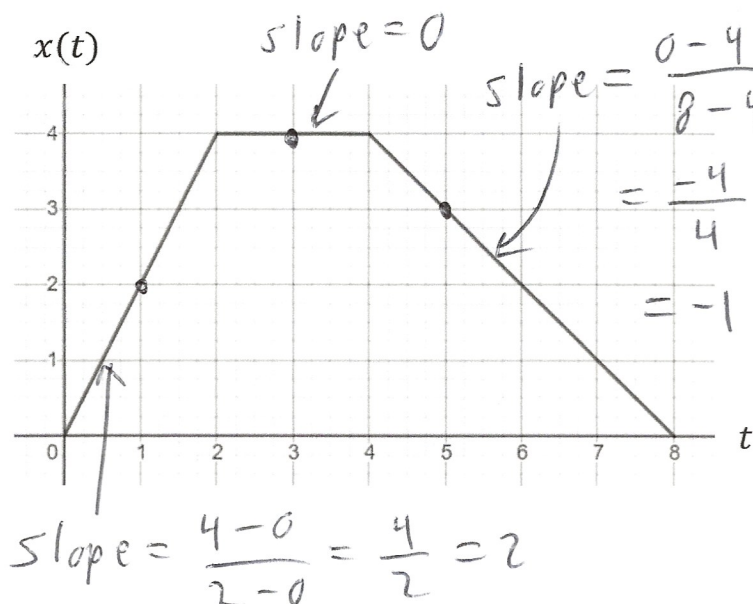
a) What is the position of the particle at 1, 3, and 5 seconds?

$$\begin{aligned} t=1 &\Rightarrow x(1)=2 \text{ meters} \\ t=3 &\Rightarrow x(3)=4 \\ t=5 &\Rightarrow x(5)=3 \end{aligned}$$

b) What is the velocity of the particle at 1, 3, and 5 seconds?

velocity = slope

velocity at $t=1$ is 2
 $t=3$ is 0
 $t=5$ is -1
meters per second



c) When is the particle at rest?

rest \Rightarrow velocity is zero \Rightarrow slope is zero

~~2 < t < 4~~
 $2 < t < 4$ seconds
at rest

d) When is the particle moving to the right?

moving right \Rightarrow velocity $> 0 \Rightarrow$ positive slope

$0 \leq t \leq 2$ seconds

AP Calculus AB – Unit 4

Dan the Tutor



Learn by Doing

8. The velocity vs. time graph of a particle is shown. The velocity is measured in m/s and the time is measured in seconds.

- a) For $0 \leq t \leq 5$, when is the particle moving to the left? When is it moving to the right?

moving left \Rightarrow velocity < 0

The velocity is always > 0
between $t=0$ and $t=5$

moving right on $[0, 5]$

never moving left

- b) When is the particle at rest?

at rest \Rightarrow velocity $= 0$

$t = 0$ seconds

- c) What is the particle's acceleration at $t = \frac{1}{2}$, $t = 2$, and $t = 4$?

acceleration = slope

acceleration at $t = \frac{1}{2}$ is 1.5

$t = 2$ is 0

$t = 4$ is $\frac{1}{2}$

$\frac{m}{s^2}$

