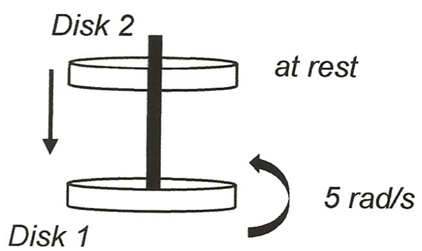
Angular Momentum Problems

1. A basketball player spins a ball on his finger with an angular velocity of 24 rad/s. If the ball's moment of inertia is 2 $kg \cdot m^2$, what is the angular momentum of the ball?

2. A solid metal disk is placed on a spoke and is allowed to rotate freely at 5 rad/s. Suddenly, a second identical disk is dropped on top of the first disk. The second disk was not rotating when it was dropped. What is the new angular velocity of the 2 disks?



3. Consider the same setup as the last problem. This time, the second disk is dropped while spinning 2.5 rad/s in the opposite direction as the first disk. What will be the new angular velocity of the 2 disks now?

4. A NASCAR driver moves towards the center of the track as he completes a turn. His turning radius changes from 75 meters to 60 meters. The car can be considered a point source of mass of 1,700 kg. If the car's tangential velocity was 64 m/s going into the turn, what is its tangential speed coming out of the turn?

5. The Earth rotates around the sun at an angular velocity of 1 revolution per year (which is $1.99 \cdot 10^{-7}$ rad/s). Then, a meteor with mass $7 \cdot 10^{16}$ kg crashes into the Earth at $5.20 \cdot 10^{11}$ m/s in the opposite direction the Earth was moving. The Earth has a mass of $6 \cdot 10^{24}$ kg and a radius around the sun of $1.5 \cdot 10^{11}$ m. If the meteor gets lodged in the middle of the Pacific Ocean, what is the new angular speed of the Earth after the collision?

6. In order to complete a pirouette, a figure skater pulls her arms in, which causes her angular velocity to triple. If her initial rotational kinetic energy was 1,000 J, what is her new kinetic energy after she pulls her arms in? (Hint: angular momentum is conserved but energy is not)

7. A tortoise is placed on a turn table, and someone starts spinning it. At first, the tortoise's rotational kinetic energy is 270 J. Then the tortoise gets scared and hides in its shell, causing its moment of inertia to decrease by 10%. When this happens, what is the tortoise's new rotational kinetic energy?

1. $L = I\omega = 2(24) = 48 \frac{kg \cdot m^2}{s}$ annoying units

2. $L_i = L_f$ Since we can't calculate the moment of inertia for both disks and they're identical, let $I_1 = I_2 = I$

$L_i = I_1\omega_1 + I_2\omega_2$

$= I(5) + I(0)$

$L_i = 5I$

$L_f = (I_1 + I_2)\omega_f$

$L_f = 2I\omega_f$

$\frac{5I}{2I} = \frac{2I\omega_f}{2I}$

$\omega_f = \frac{5}{2}$ rad/s

2.5

3. $L_i = I_1\omega_1 + I_2\omega_2$

$= I(5) + I(-2.5)$

$L_i = 2.5I$

$L_f = (I_1 + I_2)\omega_f$

$L_f = 2I\omega_f$

$L_i = L_f$

$\frac{2.5I}{2I} = \frac{2I\omega_f}{2I}$

in the same direction as the first disk

$\omega_f = 1.25 \frac{rad}{s}$

4. $L_i = L_f$ I for point source of mass $= mr^2$

$L_i = mr^2\omega$ $V = \omega r$ $\omega = \frac{V}{r}$ $L_f = mr^2\omega$

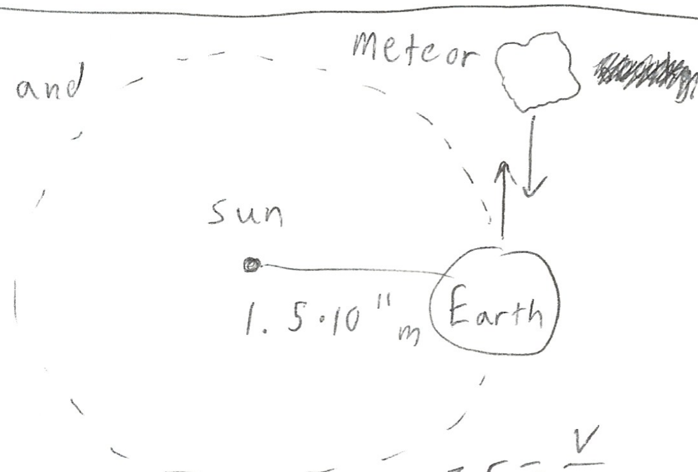
$L_i = 1700(75)^2\left(\frac{64}{75}\right)$ $= 1700(60)^2\left(\frac{V}{60}\right)$

$L_i = 8,160,000$ $L_f = 102,000\,V$

$8,160,000 = 102,000\,V$ $V = 80\ \text{m/s}$

---

5. $L_i = L_f$ Both the Earth and meteor can be treated as point sources of mass $(I = mr^2)$



stick together → perfectly inelastic

$\omega = \frac{V}{r}$

$L_i = I\omega_{Earth} + I\omega_{meteor}$

$= 6\cdot10^{24}(1.5\cdot10^{11})^2(1.99\cdot10^{-7}) + (7\cdot10^{16})(1.5\cdot10^{11})^2\left(\frac{-5.20\cdot10^{11}}{1.5\cdot10^{11}}\right)$

$= 2.6865\cdot10^{40} - 5.46\cdot10^{39}$

$L_i = 2.14\cdot10^{40}$

$L_f = (I_{Earth} + I_{meteor})\,\omega_f$

$= \left(6\cdot10^{24}(1.5\cdot10^{11})^2 + 7\cdot10^{16}(1.5\cdot10^{11})^2\right)\omega_f$

$L_f = 1.35\cdot10^{47}\,\omega_f$

$\frac{2.14\cdot10^{40}}{1.35\cdot10^{47}} = \frac{1.35\cdot10^{47}\,\omega_f}{1.35\cdot10^{47}}$

$\omega_f = 1.59\cdot10^{-7}\ \frac{rad}{s}$

6. First, use $L_i = L_f$ to find new moment of inertia

$$L_i = I_i \omega_i \qquad L_f = I_f \omega_f \leftarrow \omega_f = 3\omega_i$$

$$L_i = L_f$$

$$I_i \cancel{\omega_i} = I_f (3\cancel{\omega_i})$$

$$I_f = \frac{1}{3} I_i$$

Then, consider K_i and K_f

$$K_i = \frac{1}{2} I_i \omega_i^2 = 1{,}000 \text{ J}$$

$$K_f = \frac{1}{2} I_f \omega_f^2$$

square both 3 and ω_i

$$= \frac{1}{2}\left(\frac{1}{3} I_i\right)(3\omega_i)^2$$

$$= \frac{1}{2}\left(\frac{1}{3} I_i\right)(9\omega_i^2)$$

replace these 3 with 1,000

because $\frac{1}{2} I_i \omega_i^2 = 1000$

(the $\frac{1}{3}$ and 9 remain)

$$K_f = 1000\left(\frac{1}{3}\right)(9)$$

$$= \boxed{3{,}000 \text{ J}}$$

7. First, use $L_i = L_f$ to find new angular velocity

$L_i = L_f$

$L_f = I_f \omega_f$

Decrease by 10%

$\downarrow$

$I_f = .9 I_i$

$L_i = I_i \omega_i$

$L_f = (.9 I_i)\omega_f$

$\cancel{I_i}\omega_i = .9\cancel{I_i}\omega_f$

$\omega_f = \frac{1}{.9}\omega_i$

Then, consider K_i and K_f

$K_i = \frac{1}{2} I_i \omega_i^2 = 270$

$K_f = \frac{1}{2} I_f \omega_f^2$

$= \frac{1}{2}(.9 I_i)\left(\frac{1}{.9}\omega_i\right)^2$

Replace these with 270 because $\frac{1}{2} I_i \omega_i^2 = 270$

$= \frac{1}{2}(.9 I_i)\left(\frac{1}{.81}\omega_i^2\right)$

$K_f = 270(.9)\left(\frac{1}{.81}\right)$

$K_f = 300\ J$