Universal Gravitation Problems

For all the problems below, use the universal gravitation formula when it asks for the force of gravity.

1. What is the gravitational force between you and your friend if you are both 60 kg and you are standing 3 meters apart?

2. The Earth has a mass of $5.97 \cdot 10^{24}$ kg, and its radius to the Equator is 6,378 km. If your mass is 80 kg, what is your force of gravity using the big G (universal gravitation) formula? What is your force of gravity using the little g (9.8 m/s^2) formula?

3. Two giant asteroids head towards each other in outer space. Their masses are 30,000 kg and 75,000 kg respectively. If they are separated by a distance of 2,000 m, what is the force acting on each asteroid? What is each of their accelerations?

4. What is the potential energy due to gravity (using the universal gravitational energy equation) of a 75 kg man standing at the equator? Use the same values for mass and radius of the Earth from question 2.

5. A disgruntled student throws his calculator up into the sky. It exits Earth's atmosphere and goes into outer space. Neglecting air resistance, what velocity did the student throw their calculator? Use the same values for mass and radius of the Earth from question 2. (Hint: use conservation of energy where point 2 is infinitely far away from Earth)

6. A rocket takes off from Earth's equator with an initial speed of 8,000 m/s pointing straight towards the sky. The rocket does not have enough fuel to leave Earth's atmosphere. What final height above Earth does the rocket reach when it starts coming back down? Use the same values for mass and radius of the Earth from question 2.

1. $$F_G = G\frac{M_1 M_2}{r^2} \qquad G = 6.67\cdot 10^{-11}$$

$$= 6.67\cdot 10^{-11}\,\frac{(60)(60)}{3} = \boxed{8.00\cdot 10^{-8}\ \text{N}}$$

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2. $$r = 6{,}378\ \text{km} = 6{,}378{,}000\ \text{m}$$

Big G

$$F_G = 6.67\cdot 10^{-11}\,\frac{(5.97\cdot 10^{24})(80)}{(6{,}378{,}000)^2} = \boxed{783.1\,\text{N}}$$

Little g

$$F_g = mg$$

$$= 80(9.8)$$

$$= \boxed{784\ \text{N}}$$

Pretty darn close

3. The force of each asteroid on the other is equal because of Newton's 3rd Law.

$$F_G = 6.67 \cdot 10^{-11} \frac{(30{,}000)(75{,}000)}{(2{,}000)^2} = \boxed{3.75 \cdot 10^{-8}\ \text{N}}$$

Force of gravity
(very small number)

Finding accelerations

$$F = ma$$

<u>Asteroid 1</u>

$$3.75 \cdot 10^{-8} = 30{,}000\, a$$

$$\boxed{a = 1.25 \cdot 10^{-12}\ \text{m/s}^2}$$

<u>Asteroid 2</u>

$$3.75 \cdot 10^{-8} = 75{,}000\, a$$

$$\boxed{a = 5.00 \cdot 10^{-13}\ \text{m/s}^2}$$

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4. $$U_G = -G\frac{Mm}{r}$$

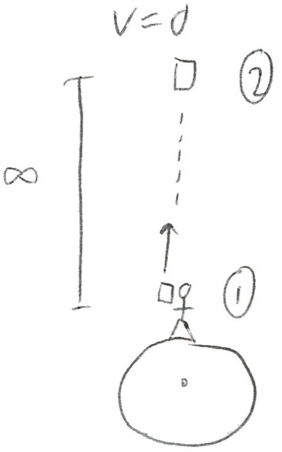
6,378 km
= 6,378,000 m

$$= -6.67 \cdot 10^{-11} \frac{(5.97 \cdot 10^{24})(75)}{6{,}378{,}000}$$

$$\boxed{= -4.68 \cdot 10^{9}\ \text{J}}$$

Note that universal gravitational energy will always be negative

5.



$$E_{tot,1} = E_{tot,2}$$

$$U_G + K = 0$$

U_G must be zero

because $r = \infty$ and

$$U_G = -\frac{GMm}{\infty} = 0$$

kinetic energy must be zero because point 2 is the peak height reached.

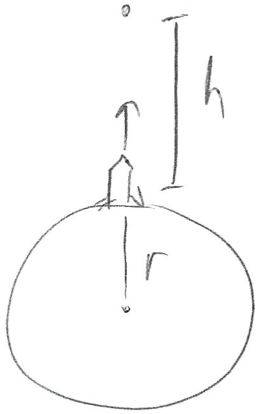
$$-G\frac{M_{Earth}\, m_{calc}}{r_{Earth}} + \frac{1}{2} m_{calc} v^2 = 0$$

$$-\frac{6.67\cdot10^{-11}(5.97\cdot10^{24})}{6{,}378{,}000} m_{calc} + \frac{1}{2} m_{calc} v^2 = 0$$

Add that to both sides

$$\frac{1}{2}\cancel{m_{calc}} v^2 = 6.67\cdot10^{-11}\,\frac{5.97\cdot10^{24}}{6{,}378{,}000}\,\cancel{m_{calc}}$$

$$\frac{1}{2}v^2 = 6.24\cdot10^{7} \qquad v^2 = 1.25\cdot10^{8} \qquad \boxed{v = 11{,}174\ \text{m/s}}$$



6. $E_{tot,1} = E_{tot,2}$

$$U_{G,1} + K_1 = U_{G,2} + K_2$$

K_2 = 0 because peak height

$$-G\frac{M_{Earth}\cancel{m_{rocket}}}{r_{Earth}} + \frac{1}{2}\cancel{m_{rocket}}V^2 = -G\frac{M_{Earth}\cancel{m_{rocket}}}{r_{Earth}+h}$$

$$-6.67\cdot10^{-11}\,\frac{5.97\cdot10^{24}}{6,378,000} + \frac{1}{2}(8,000)^2 = -6.67\cdot10^{-11}\,\frac{5.97\cdot10^{24}}{6,378,000+h}$$

$$-3.043\cdot10^{7} = \frac{-3.982\cdot10^{14}}{6,378,000+h}$$

$$(6,378,000+h)(-3.043\cdot10^{7}) = -3.982\cdot10^{14}$$

Divide both sides by $-3.043\cdot10^{7}$

$$6,378,000+h = 1.308\cdot10^{7}$$

$$\boxed{h = 6.71\cdot10^{6}\text{ m}}$$

← Final height above Earth's surface