Problem Set 15: Inverse Trig Derivatives

Find the derivative of the inverse trig functions.

1. $y = \cos^{-1}(3x - 3)$ $u = 3x-3$

$u' = 3$

$$y' = \frac{-u'}{\sqrt{1-u^2}}$$

$$y' = \frac{-3}{\sqrt{1-(3x-3)^2}}$$

2. $y = \csc^{-1}(4x^2)$ $u = 4x^2$ $u' = 8x$

$$y' = \frac{-u}{|u|\sqrt{u^2-1}}$$

$$y' = \frac{-8x}{|4x^2|\sqrt{(4x^2)^2-1}}$$

always positive

$$= \frac{-\cancel{8}^{2}\cancel{x}}{\cancel{4}x^{\cancel{2}}\sqrt{16x^4-1}} = \frac{-2}{x\sqrt{16x^4-1}}$$

3. $y = \dfrac{\sin^{-1}(x)}{x}$

$u = \sin^{-1}(x)$ $v = x$

$u' = \dfrac{1}{\sqrt{1-x^2}}$ $v' = 1$

$$y' = \frac{x\left(\frac{1}{\sqrt{1-x^2}}\right) - \sin^{-1}(x)(1)}{x^2}$$

$$= \frac{\frac{x}{\sqrt{1-x^2}} - \sin^{-1}(x)}{x^2}\left(\frac{\sqrt{1-x^2}}{\sqrt{1-x^2}}\right)$$

$$= \frac{x - \sin^{-1}(x)\sqrt{1-x^2}}{x^2\sqrt{1-x^2}}$$

4. $y = \tan^{-1}(9x^5 - x^3 - 2x)$

$u = 9x^5 - x^3 - 2x$

$u' = 45x^4 - 3x^2 - 2$

$$y' = \frac{u'}{u^2+1}$$

$$y' = \frac{45x^4 - 3x^2 - 2}{(9x^5 - x^3 - 2x)^2 + 1}$$

5. $y = \cot^{-1}(x(2x-4))$

$y = \cot^{-1}(2x^2 - 4x)$

$u = 2x^2 - 4x$

$u' = 4x - 4$

$y' = \dfrac{-(4x-4)}{(2x^2-4x)^2+1}$

6. $y = x \cdot \sec^{-1}(x+7)$ Product Rule

$u = x \quad v = \sec^{-1}(x+7)$

$u' = 1 \quad v' = \dfrac{1}{|x+7|\sqrt{(x+7)^2-1}}$

$y' = \dfrac{x}{|x+7|\sqrt{(x+7)^2-1}} + \sec^{-1}(x+7)$

7. $y = \csc^{-1}(\tan x)$

$u = \tan x$

$u' = \sec^2 x$

$y' = \dfrac{-u'}{|u|\sqrt{u^2-1}}$

$\cancel{y' = -\tan x}$

$y' = \dfrac{-\sec^2 x}{|\tan x|\sqrt{\tan^2 x - 1}}$

8. $y = \cos^{-1}\left(\dfrac{1}{2x^3}\right)$ $u = (2x^3)^{-1}$

chain rule $\rightarrow$ $u' = -(2x^3)^{-2}(2(3x^2))$

$y' = \dfrac{-u'}{\sqrt{1-u^2}}$

$u' = \dfrac{-6x^2}{(2x^3)^2}$

$u' = \dfrac{-\cancel{6}^{3}\cancel{x^2}}{2\cancel{4}x^{\cancel{6}4}}$

$u' = \dfrac{-3}{2x^4}$

$y' = \dfrac{-\frac{-3}{2x^4}}{\sqrt{1-\left(\frac{1}{2x^3}\right)^2}}$

$= \dfrac{3}{2x^4\sqrt{1-\frac{1}{4x^6}}}$