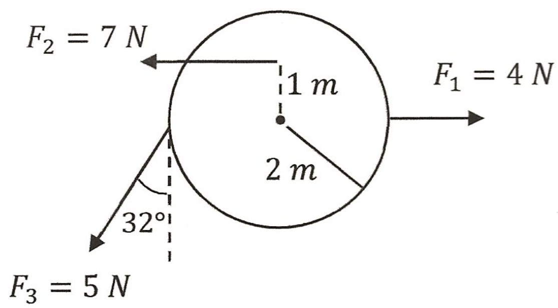
Dan the Tutor

Learn by Doing

Torque and Moment of Inertia Practice Problems

1. You open a door by pulling with a force of 20 N perpendicular to the door. The distance between the door handle and the hinge is 80 cm. How much torque did you perform?

2. The following forces are performed on the wheel. What is the magnitude and direction of the net torque?



3. On a playground, there's a spinning wheel (like a merry-go-round) that can be thought of as a solid disk. The wheel has a mass of 150 kg and a radius of 1.25 meters. A 30 kg kid (which can be thought of as a point source of mass) jumps on the edge of the wheel. What is the total moment of inertia of the kid and the wheel?

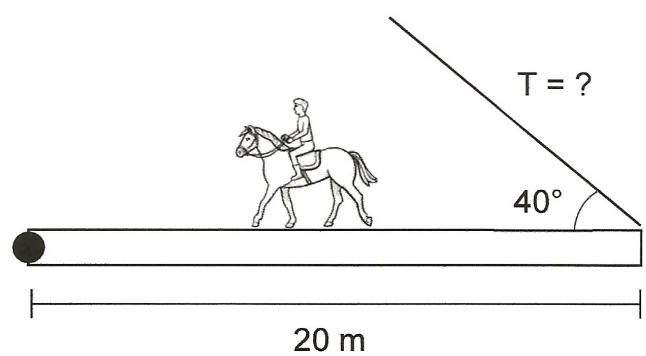
4. A 1 meter barbell has 50 kg plates on either end. The bar itself has a mass of 25 kg. What is the moment of inertia when the pivot point is at the center of the bar? When the pivot point is at one end of the bar?

5. A 30 kg spool of thread with radius 26 cm is pulled with a force of 120 N, causing it to start spinning. The spool can be thought of as a solid disk. What will be the angular velocity of the spool after 10 seconds?

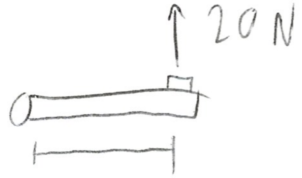
6. A man applies a 150 N force to a 1.2 kg wrench at an angle of 75° to the handle. If the wrench is 20 cm long, and the wrench is approximated as a thin rod hinged at one end, what is the angular acceleration of the wrench?

7. A 2-meter-long plank of wood is resting on a balance beam with the fulcrum in the middle. A 15 kg block is placed 35 cm left of the fulcrum. How far to the right of the fulcrum must a 21 kg block be placed to balance the weight?

8. A 20-meter-long draw bridge is held up by a rope angled at 40° above the bridge while the other end is fixed in place. The bridge has a mass of 1000 kg. A man on a horse (mass 160 kg) walks across the bridge. When the rider is at the center of the bridge, what is the tension in the rope?



1.



80 cm = .8 m

$\tau = F \cdot d \sin\theta$

$\sin\theta$ → pull perpendicular ↓ $\sin 90° = 1$

$\tau = 20(.8)$

$= 16 \text{ N·m}$

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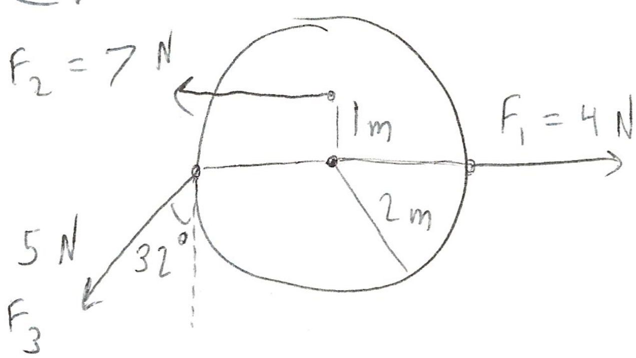
2. $\tau = F \cdot d \sin\theta$



$\tau_1 = 4(2)\sin 0°$

↑ 0° because F and d are parallel

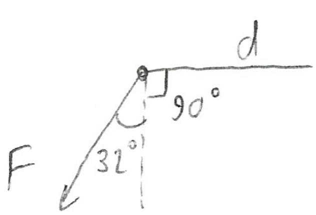
$\tau_1 = 0$

$\tau_2 = 7(1)\sin 90°$

↑ perpendicular

$\tau_2 = 7 \text{ N·m}$

$\tau_3 = 5(2)\sin(122°)$



$32 + 90 = 122°$

$\tau_3 = 8.48 \text{ N·m}$

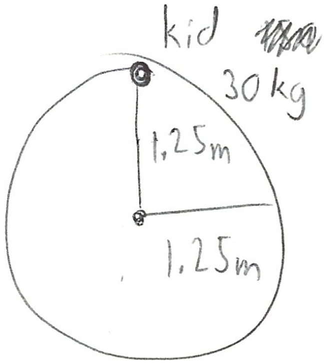
Net Torque $= \tau_1 + \tau_2 + \tau_3$

$= 0 + 7 + 8.48$

↑ ↑ both point counter-clockwise, so both positive

Net torque → 15.48 N·m

3.



$$I_{tot} = I_{wheel} + I_{kid}$$

$$I_{wheel} = \frac{1}{2} m r^2 = \frac{1}{2}(150)(1.25)^2 = 117.2$$

(I_{wheel} ← disk)

$$I_{kid} = m r^2 = 30(1.25)^2 = 46.9$$

(I_{kid} ← point source of mass)

$$I_{tot} = 117.2 + 46.9 = \boxed{164.1\ kg \cdot m^2}$$

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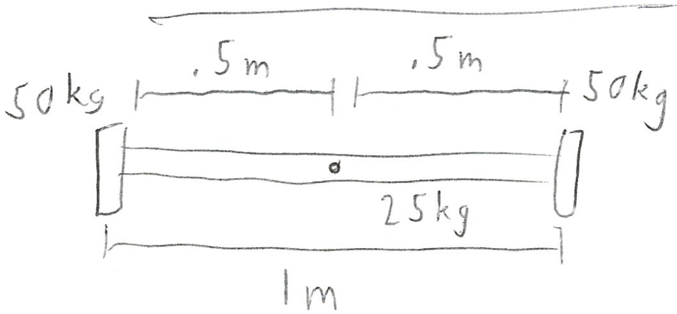
4. Pivot point at center



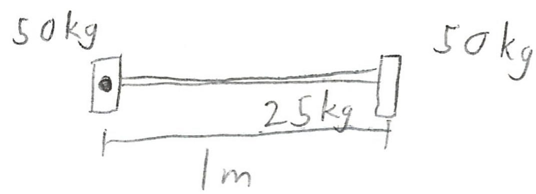
$$I_{tot} = I_{point\ source} + I_{rod,center} + I_{point\ source}$$

$$\phantom{I_{tot}} \quad\ \ m r^2 \qquad\quad \frac{1}{12} m L^2 \qquad\quad m r^2$$

$$= 50(.5)^2 + \frac{1}{12}(25)(1)^2 + 50(.5)^2$$

$$= \boxed{27.1\ kg \cdot m^2}$$

Pivot point at end



$$I_{tot} = I_{rod,end} + I_{point\ source}$$

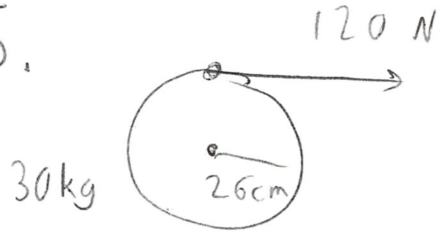
$$\qquad\quad \frac{1}{3} m L^2 \qquad\quad m r^2$$

$$= \frac{1}{3}(25)(1)^2 + 50(1)^2$$

$$= \boxed{58.3\ kg \cdot m^2}$$

(The left plate is on the pivot point, so it's I is zero)

5.



26 cm = .26 m

$I_{spool} = \frac{1}{2} m r^2 = \frac{1}{2}(30)(.26)^2 = 1.014$

disk (↑ spool)

$\tau = F \cdot d \sin\theta = 120(.26)\sin 90^\circ = 31.2$

(↑ 90°: perpendicular)

$\tau = I\alpha$

$31.2 = 1.014\alpha$

$\alpha = 30.77$

To find angular velocity, use angular kinematics

$\omega_i = 0$ ← starts from rest

$\omega_f = ?$

$\alpha = 30.77$

$t = 10$

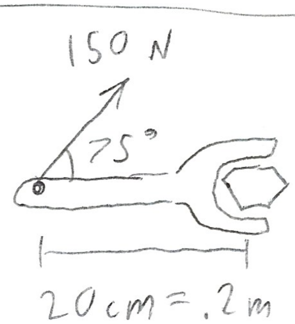
$\Delta\theta$

$\omega_f = \omega_i + \alpha t$

$\omega_f = 0 + 30.77(10)$

$\omega_f = 307.7$ rad/s

6.



$I_{wrench} = \frac{1}{3} m L^2 = \frac{1}{3}(1.2)(.2)^2$

thin rod at end (↑ wrench)

$I = .016\ kg \cdot m^2$

$\tau = F d \sin\theta$

$= 150(.2)(\sin 75^\circ)$

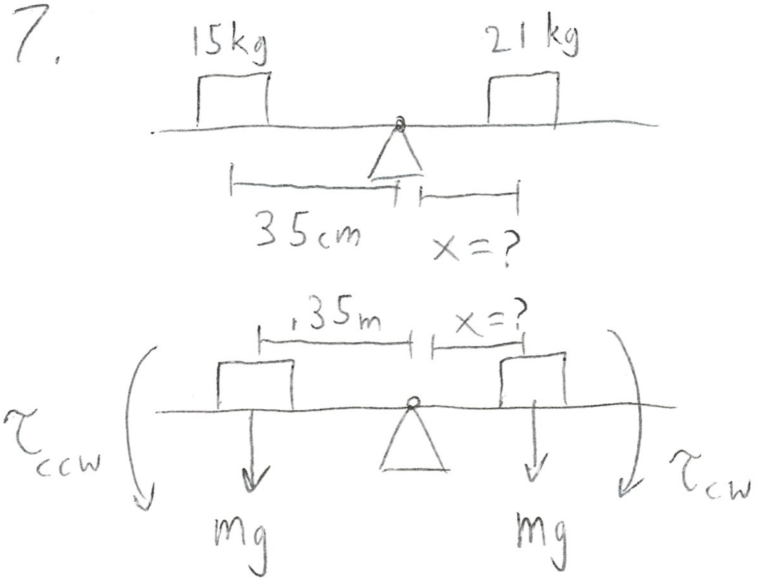
$\tau = 28.98\ N \cdot m$

$\tau = I\alpha$

$28.98 = .016\alpha$

$\alpha = 1811.1$ rad/s^2 ← angular acceleration

7.



$\tau = F \cdot d \sin\theta$

$\tau_{ccw} = \tau_{cw}$

(rotational static equilibrium)
↑
not rotating or moving

$\tau_{ccw} = mg \cdot d \sin\theta$

$= 15(9.8)(.35)\sin 90^\circ$

$\tau_{ccw} = 51.45 \text{ N} \cdot \text{m}$

$\tau_{cw} = mg \cdot d \sin\theta$

$= 21(9.8)(x)\sin 90^\circ$

$= 205.8x$

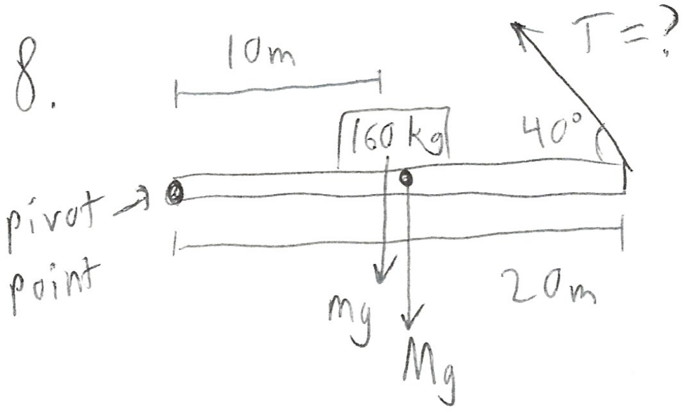
$\tau_{ccw} = \tau_{cw}$

$51.45 = 205.8x$

$x = .25 \text{ m}$ or 25 cm

The 21 kg block should be placed 25 cm to the right of the fulcrum.

8.



Rotational static equilibrium

↓

nothing is moving or rotating

↓

torques must cancel

$$\tau_{horse} + \tau_{bridge} = \tau_{rope}$$

torques point in same direction

$$mgd + Mgd = Td\sin\theta$$

$$160(9.8)(10) + 1000(9.8)(10) = T(20)\sin 40^\circ$$

The bridge must be treated as a single mass at its center

$$15{,}680 + 98{,}000 = 12.86\,T$$

$$113{,}680 = 12.86\,T$$

$$T = 8{,}842.7\text{ N}$$

tension in the rope