Practice Problems | Extrema and Other Applications of Derivatives

Absolute Extrema Practice

1. Find the absolute extrema for $f(x) = x^2$ on [-2,3]
2. Find all extrema for $f(x) = \sqrt[3]{x}$ on [-1,2]
3. Find all extrema for $f(x) = \sqrt{x}$ on [4,9)

Relative Extrema Practice

4. Find all extrema for $f(x) = x^3 - \frac{3}{2}x^2$
5. Find all extrema for $f(x) = \frac{x^4+1}{x^2}$
6. Find all extrema for $f(x) = \frac{1}{\sqrt[3]{(x^2-1)}}$

Second Derivative Test Practice

7. Find all local extrema for $f(x) = -\frac{2}{3}x^3 - x^2 + 5$
8. Find all extrema for $f(x) = 2x^4 - 4x^2 + 1$
9. Find all local extrema for $f(x) = x^3 - 2x^2 - 1$

Mean Value Theorem Practice

10. Determine if $f(x) = 3x^3 - 2x$ satisfies the hypothesis of the MVT on [0,1]. If so, find all values of c that satisfy that theorem. If not, explain your reasoning.
11. Determine if $f(x) = x^{\frac{2}{3}}$ satisfies the hypothesis of the MVT on [0,1]. If so, find all values of c that satisfy the theorem. If not, explain your reasoning.
12. Determine if $f(x) = |x - 1|$ satisfies the hypothesis of the MVT on [0,4]. If so, find all values of c that satisfy the theorem. If not, explain your reasoning.

Optimization Practice

Find two positive numbers that satisfy the given requirements

13. The product is 192 and the sum is a minimum

14. The product is 192 and the sum of the first plus three times the second is a minimum

15. The second number is the reciprocal of the first and the sum is a minimum

16. The sum of the first and twice the second is 100 and the product is a maximum

Find Perimeter and Area

17. Find the length and width of a rectangle that has a perimeter of 100 meters and a maximum area

18. Find the length and width of a rectangle that has an area of 64 square feet and a minimum perimeter

Word Problems

19. A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 180,000 square meters in order to provide enough grass for the herd. What dimensions would require the least amount of fencing if no fencing is needed along the river?

20. A rancher has 200 feet of fencing with which to enclose two adjacent rectangular corrals. What dimensions should be used so that the enclosed area is a maximum?

21. A Norman window is when a semicircle is added to an ordinary rectangular window. Find the dimensions of a Norman window of a maximum area if the total perimeter is 16 feet.

22. A rectangle is bounded by the x-axis and the semicircle $y = \sqrt{25 - x^2}$. What length and width should the rectangle have so that area is a maximum?

23. A rectangular page needs 30 square inches of print. Each side are to be 1 inch. Find the dimensions of the page such that the least paper is used

24. A rectangular page is to contain 36 square inches of print. The margins on each side are to be 1.5 inches. Find the dimensions of the page such that the last amount of paper is used.

25. A solid is formed by adjoining two hemispheres to the ends of a right circular cylinder. The total volume of the solid is 12 cubic centimeters. Find the radius of the cylinder that produces the minimum surface area.

26. An industrial tank of the shape described in problem 25 must have a volume of 3000 cubic feet. The hemispherical ends cost twice as much per square foot of surface area as the sides. Find the dimensions that will minimize cost.

Find the point on the graph of the function that is closest to the given point

27. Function: $f(x) = \sqrt{x}$
    Point: (4,0)

28. Function: $f(x) = \sqrt{x - 8}$
    Point: (2,0)

29. Function: $f(x) = x^2$
    Point: (2,1/2)

30. Function: $f(x) = (x + 1)^2$
    Point: (5,3)

Related Rates

31. The radius r of a circle is increasing at a rate of 3 centimeters per minute. Find the rate of change of the area when r=6cm and r=12cm

32. A spherical balloon is inflated with gas at the rate of 800 cubic centimeters per minute. How fast is the radius of the balloon increasing at the instant the radius is 30 centimeters and 60 centimeters?

33. All edges of a cube are expanding at a rate of 3 centimeters per second. How fast is the volume changing when each edge is 1 centimeter and 10 centimeters?

34. The conditions are the same as problem 33. Determine how fast the surface area is changing when each edge is 1 centimeter and 10 centimeters?

35. Sand is falling off a conveyor and onto a conical pile at a rate of 10 cubic feet per minute. The diameter of the base of the cone is approximately three times the altitude. At what rate is the height of the pile changing when the pile is 15 feet high?

36. A conical tank with vertex down is 10 feet across the top and 12 feet deep. If water is flowing into the tank at a rate of 10 cubic feet per minute, find the rate of change of the depth of the water when the water is 8 feet deep.

37. An airplane is flying at an altitude of 5 miles and passes over a radar antenna. When the plane is 10 miles away (s=10) , the radar detects that the distance is changing at 240 mph. What is the speed of the plane?

38. An air traffic controller spots two planes at the same altitude converging on a point as they fly at right angles to each other. One plane is 150 miles from the point moving at 450 miles per hour. The other plane is 200 miles from the point moving at 600 miles per hour. At what rate is the distance between the planes decreasing? How much time does the air traffic controller have to get one of the planes on a different flight path?

39. A baseball diamond has the shape of a square with sides 90 feet long. A player running from second base to third base at a speed of 28 feet per second is 30 feet from third base. At what rate is the player's distance s from home plate changing?

40. For the baseball diamond in problem 39, suppose the player is running from first to second at a speed of 28 feet per second. Find the rate at which the distance from home plate is changing when the player is 30 feet from second base.

41. A man 6 feet tall walks at a rate of 5 feet per second away from a light that is 15 feet above the ground. When he is 10 feet from the base of the light, what rate is the tip of his shadow moving? At what rate is the length of his shadow changing?

42. Repeat problem 41 for a man 6 feet tall walking at a rate of 5 feet per second toward a light that is 20 feet above the ground.

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Linearization and Error Practice

43. Use a linearization to approximate $(3.01)^4$

44. The area of a square was measured to be 25 square centimeters, with a possible error 0.12 square centimeters. Find the percent error in measuring the side.

45. Use a linearization to approximate $\sqrt{50}$

46. The area of a circle was measured to be 16 square centimeters, with a possible error of 0.1 square centimeters. Find the percent error in measuring the radius.

L'Hopitals Rule Practice

47. $\lim_{x\to 0^+} \frac{sinx}{x^2}$

48. $\lim_{x\to \infty} \frac{x}{e^x}$

49. $\lim_{x\to \infty} \frac{e^x}{x^3}$

50. $\lim_{x\to \pi} (x-\pi)cotx$

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Answer Key

1. Absolute maximum: (3,9)
   Absolute minimum: (0,0)
2. Absolute maximum: $(2,\sqrt[3]{2})$
   Absolute minimum: (-1,-1)
3. Absolute maximum: (4,2)
   No Absolute minimum
4. Local maximum: x=0
   Local minimum: x=1
5. No Local maximum
   Local minimum: x=-1,1
6. Local maximum: x=0
   No Local minimum
7. Local maximum: x=0
   Local minimum: x=-1
8. Local maximum: x=0
   Local minimum: x=-1,1
9. Local maximum: x=0
   Local minimum: x=4/3
10. $c = \frac{\sqrt{3}}{3}$
11. $c = \frac{8}{27}$
12. Since f(x) is not differentiable at x=1, the theorem does not apply
13. $\sqrt{192}, \sqrt{192}$
14. 8, 24
15. 1, 1
16. 50, 25
17. L=W=25m
18. L=W=8ft
19. 600 x 300m
20. X = 25ft, Y = 100/3ft
21. $\frac{16}{\pi+4} x \frac{32}{\pi+4}$ft
22. $x = 3, y = \frac{3}{2}$
23. $(2+\sqrt{30}) in\ x\ (2+\sqrt{30}) in$
24. 9 x 9
25. 1.4202 cm
26. r = 5.6362
27. $(\frac{7}{2}, \frac{\sqrt{14}}{2})$
28. $(8,0)$
29. (1,1)
30. (1,4)

Answer Key

31. $36\pi\ cm^2/min$ and $144\pi\ cm^2/min$
32. $\frac{2}{9\pi}\ cm/min$ and $\frac{1}{18\pi} cm/min$
33. $9\frac{cm^3}{s}$
34. $36\frac{cm^2}{s}\ and \frac{360cm^2}{s}$
35. $\frac{9}{10\pi}\ ft/min$
36. $\frac{8}{405\pi} ft/min$
37. $277.1281\ mph$
38. -750 mph, 20 min
39. -8.8545 ft/s
40. 15.5316 ft/s
41. $\frac{25}{3} ft/s$ and $\frac{10}{3} ft/s$
42. $-\frac{50}{7} ft/s$ and $-\frac{15}{7} ft/s$
43. $f(3.01) \approx 82.08$
44. $\pm 0.24\%$
45. $f(50)\ \approx \frac{99}{14}$
46. $\pm 0.312\%$
47. $\infty;\ Does\ not\ exist$
48. 0
49. $\infty; Does\ not\ exist$
50. 1