Unit 4 Exam

Recommended amount of time: 2 hours 30 mins

- Topics covered on this exam:
  - Critical values
  - Particle motion
  - Concavity and points of inflection
  - Relative and absolute extrema
  - First and second derivative tests
  - Mean Value Theorem
  - Optimization and related rates
  - Local linear approximation (linearization)
  - L'Hospital's Rule
- This exam is composed of 20 multiple choice questions and 4 free response questions
- Calculators are allowed for computation use only (addition, division, square root, exponents, etc.)
- For the free response section, show all of your work (think about partial credit on an actual test)
- When you are finished with the test, see which questions you got wrong and review those questions

1. $\lim_{x\to 2}\left(\frac{x^2+x-6}{x^2-4}\right)$ is

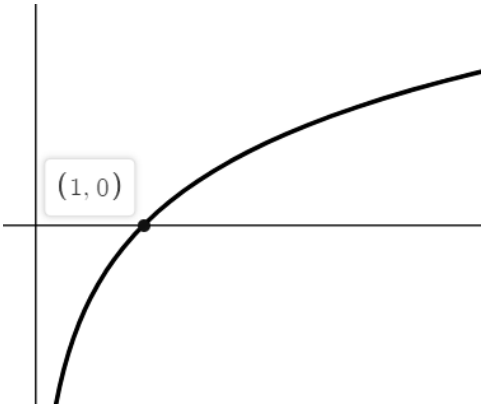
(A) $\frac{1}{4}$ (B) -1 (C) $\frac{5}{4}$ (D) 1 (E) $\frac{3}{2}$

2. If $f(x)=\sin\left(\frac{x}{2}\right)$, then there exists a number c in the interval $\frac{\pi}{2}\le x\le\frac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be c?

(A) $\frac{2\pi}{3}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{6}$ (D) π (E) $\frac{3\pi}{2}$

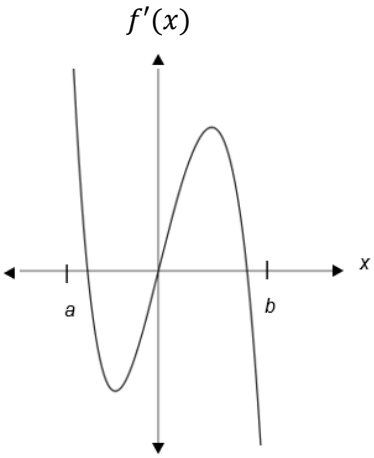
3. At what value does the graph of $y=\frac{12}{5}x^5-3x^2-37x$ have a point of inflection?

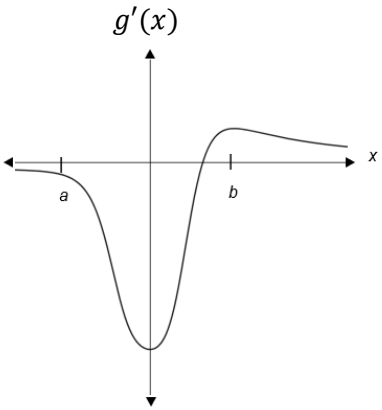
(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) $\frac{1}{3}$ (E) At no value of x

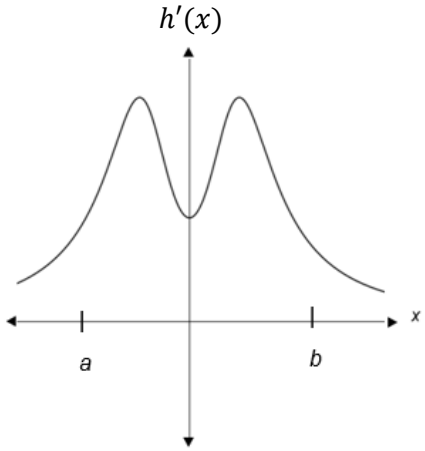


4. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f(1) < f'(1) < f''(1)$
(B) $f(1) < f''(1) < f'(1)$
(C) $f'(1) < f(1) < f''(1)$
(D) $f''(1) < f(1) < f'(1)$
(E) $f''(1) < f'(1) < f(1)$







5. The graphs of the derivatives of the functions $f, g, and\ h$ are shown above. Which of the functions $f, g, or\ h$ have a relative maximum on the open interval $a < x < b$?

(A) f only
(B) g only
(C) h only
(D) f and g only
(E) f, g, and h

6. For what value of x does the function $f'(x) = (x-1)^2 - 4$ have a relative maximum?

(A) -1 (B) $-\frac{7}{3}$ (C) $-\frac{5}{2}$ (D) $\frac{7}{3}$ (E) $\frac{5}{2}$

7. How many critical points does the function $f(x) = \frac{x}{2x^2+2}$ have?

(A) One (B) Two (C) Three (D) Five (E) Nine

8. The function f given by $f(x) = x^3 + 12x - 24$ is
(A) Increasing for $x < -2$, decreasing for $-2 < x < 2$, increasing for $x > 2$
(B) Decreasing for $x < 0$, increasing for $x > 0$
(C) Increasing for all x
(D) Decreasing for all x
(E) Decreasing for $x < -2$, increasing for $-2 < x < 2$, decreasing for $x > 2$

9. The maximum velocity attained on the interval $0 \leq t \leq 3$ by the particle whose position is given by $p(t) = t^3 - 3t^2 + 12t + 4$ is
(A) 9 (B) 12 (C) 14 (D) 21 (E) 40

10. The base b of a triangle is 3 inches and increasing at a rate of 4 inches per minute while its height h is 5 inches and decreasing at a rate of 2 inches per minute. What is the rate of change of the area of the triangle?

(A) The area is increasing by $7\ in^2/min$
(B) The area is decreasing by $6\ in^2/min$
(C) The area is decreasing by $12\ in^2/min$
(D) The area is increasing by $14\ in^2/min$
(E) The area is increasing by $3\ in^2/min$

11. The radius of a circle is decreasing at a constant rate of 0.2 centimeter per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?

(A) $-(0.4)\pi C$
(B) $-(0.2)C$
(C) $-\frac{(0.1)C}{\pi}$
(D) $(0.2)^2C$
(E) $(0.2)^2\pi C$

12. Let f be a differentiable function such that $f(3) = 2$ and $f'(3) = 5$. If the tangent line to a graph of f at $x = 3$ is used to find an approximation to a zero of f, that approximation is

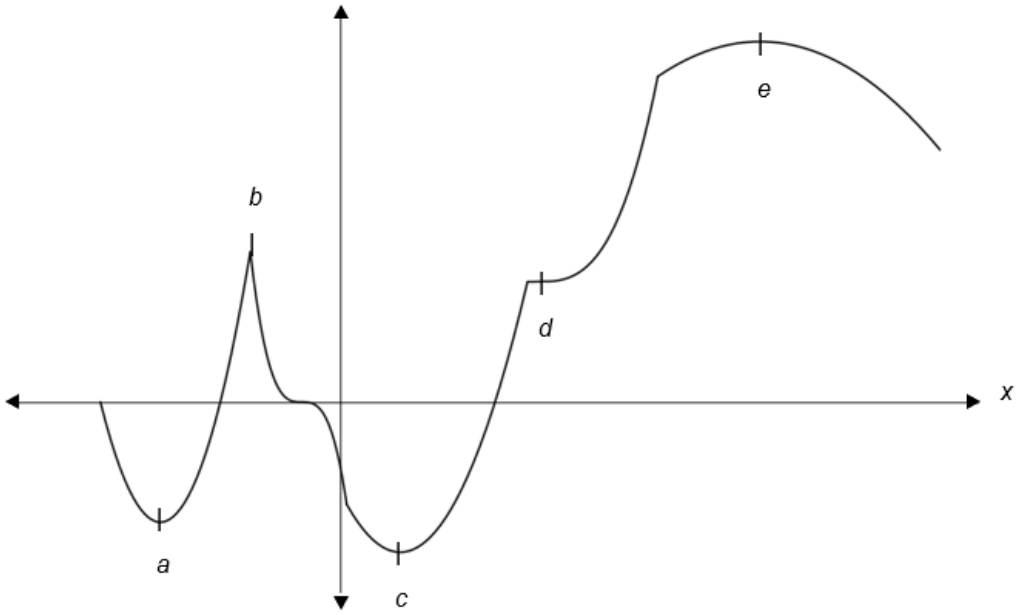
(A) 0.4 (B) 0.5 (C) 2.6 (D) 3.4 (E) 5.5

13. For small values of h, the function $\sqrt[4]{16+h}$ is best approximated by which of the following?

(A) $4 + \frac{h}{32}$ (B) $2 + \frac{h}{32}$ (C) $\frac{h}{32}$ (D) $4 - \frac{h}{32}$ (E) $2 - \frac{h}{32}$

14. Find the x value for the absolute maximum of the function $f(x) = -x^3 - 3x^2 + 9x + 12$ over the interval $[-4,2]$?

(A) 2 (B) 1 (C) 0 (D) -1 (E) -2



15. Above is a graph of $f(x)$. How many relative minimums are shown in the figure above?
(A) None (B) One (C) Two (D) Three (E) Four

16. Let f be a twice differentiable function, and let $f(4) = 2$, $f'(4) = 0$, and $f''(4) = 0$. What occurs in the graph of f at the point $(4,2)$?
(A) f is increasing at the point $(4,2)$
(B) f is decreasing at the point $(4,2)$
(C) $(4,2)$ is a relative maximum
(D) $(4,2)$ is a relative minimum
(E) There is not enough information to tell

17. Let f be a twice differentiable function, and let $f(1) = -2$, $f'(1) = 0$, and $f''(1) = -3$. What occurs in the graph of f at the point $(1, -2)$?

(A) f is increasing at the point $(1, -2)$
(B) f is decreasing at the point $(1, -2)$
(C) $(1, -2)$ is a relative maximum
(D) $(1, -2)$ is a relative minimum
(E) There is not enough information to tell

18. Let f be a differentiable function such that $f(1) = 2$ and $f'(1) = 3$. If the tangent line to the graph of f at $x = 3$ is used to find the approximation to f at five, that approximation is

(A) 11 (B) 12 (C) 13 (D) 14 (E) 15

19. The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for

(A) $x < 0$

(B) $x > 0$

(C) $x < -2$ or $x > -\frac{2}{3}$

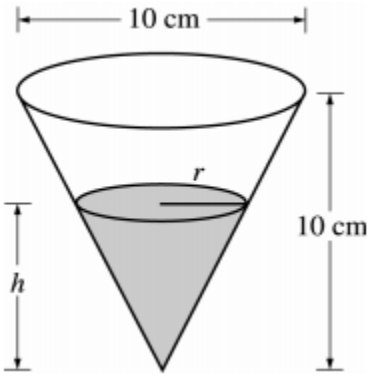
(D) $x < \frac{2}{3}$ or $x > 2$

(E) $\frac{2}{3} < x < 2$

20. The graph of the function $y = x^3 - 6x^2 + 4x - 11$ changes concavity at $x =$

(A) 11 (B) 2 (C) -11 (D) 0 (E) $\frac{3}{2}$

Free Response



1. A container has the shape of an open right circular cone, as shown in the figure to the right. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $-\frac{1}{10}$ cm/hr. (The volume of a cone can be solved using $V = \frac{1}{3}\pi r^2 h$)

a. Find the radius r of the water in the container when $h = 5$ cm. Indicate units of measure.

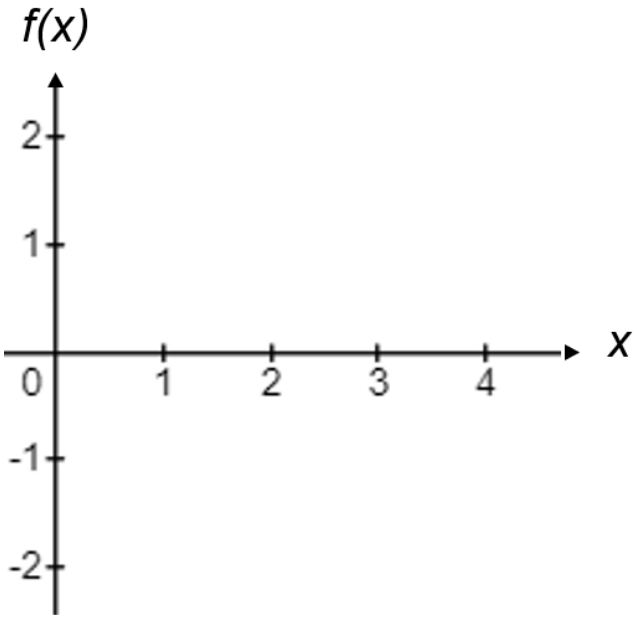
b. What is the rate of change of the radius of water in the container, with respect to time, when $h = 5$ cm? Indicate units of measure. Hint: remember that the water level h is changing at the constant rate of $-\frac{1}{10}$ cm/hr.

c. What is the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm? Indicate units of measure.

2. Let f be a function that is even and continuous on the closed interval $[0,4]$. The function f and its derivatives have the properties indicated in the table below.

x	0	0<x<1	1	1<x<2	2	2<x<3	3	3<x<4	4
$f(x)$	1	Positive	0	Negative	-1	Negative	0	Positive	1
$f'(x)$	0	Negative	-1	Negative	0	Positive	1	Positive	0
$f''(x)$	-1	Negative	0	Positive	1	Positive	0	Negative	-1

a) In the xy-plane provided below, sketch the graph of the function with all the given characteristics of f.



b) Find the x-coordinate of each point where f has a relative maximum or relative minimum on the interval $0 \leq x \leq 4$. Justify your answer.

c) Find the x-coordinate of each point of inflection on the graph of f. Justify your answer.

3. A rectangular garden is to be constructed using a rock wall as one side of the garden and wire fencing for the other three sides. Given $100\ ft$ of wire fencing, determine the dimensions that would create a garden of maximum area. What is the maximum area you can make?

4. Determine if the Mean Value Theorem can be applied to the following function on the given closed interval. If so, find all values of c: $f(x) = \frac{x}{1+x}$ on $[1,3]$.